QUADRIVIUM

Assignment 9b

The Multitudes: Arithmetic & Music

-Annandale- Tuesday, April 4nd ***-Fishkill- Wednesday, April 11th***

Read the following and do the homework exercises below (Turn them in on Thursday):
Newsome, Daniel. Excerpts from *Quadrivial Pursuits*. (2011) Read pp. 1-10 [General Introduction, Quadrivial Arithmeitc, and Quadrivial Music]- Handout and in Volume 2 of the reader posted online.

root	2	3	4	5	6	7	8	9	10	11	12	13
square	4	9	16	25	36	49	64	81	100	121	144	169

a. $6^2 - 5^2 = 11 = 1(6+5)$	Find the pattern in these 4 equations. These are described by Boethius on pp. 56-57. I translated them into algebraic notation. In the following problems, figure out what the variables are based on the patterns you found in the 4 to the left.
b. $9^2 - 7^2 = 32 = 2(9+7)$	
c. $13^2 - 10^2 = 69 = 3(13+10)$	
d. $10^2 - 6^2 = 64 = 4(10+6)$	

1. $34^2 - 33^2 = x = y(34+33)$ What is x? What is y?

2. $13^2 - 11^2 = x = y(z+w)$ What are x, y, z, and w?

3. $15^2 - 12^2 = x = y(z+w)$ What are x, y, z, and w?

4. $17^2 - 13^2 = x = y(z+z-y)$ What are x, y, and z?

5. $s^2 - t^2 = x = y(16+14)$ What are x, y, s, and t?

6. $(x+6)^2 - (x)^2 = y = z(9+3)$ What are x, y, and z?

7. $2^2 - 0^2 = x = y(2+0)$ What are x and y?

8. $2^2 - (-3)^2 = x = y[2+(-3)]$ What are x and y?

A modern major scale is made up the following intervals. [W=whole step and h=half step]

Do - re - mi-fa - sol - la - ti-do
W - W - h - W - W - W - h

It's made up of whole steps and half steps.

But premodern intervals were based on superparticular ratios and perfect fifths [3/2], forths [4/3] and octaves [2/1].

E.g. *Ptolemy's Diatonic Syntonon* is very close to our modern major scale, but instead of whole steps and half steps, it is constructed from Tones and semitones: 9/8 and 256/243. This allows a tuning to have lots of perfect fifths, fourths, and and a perfect octave.

$$\frac{9}{8}\cdot\frac{9}{8}\cdot\frac{256}{243} = \frac{3^2}{2^3}\cdot\frac{3^2}{2^3}\cdot\frac{2^8}{3^5} = \frac{2^8 3^4}{2^6 3^5} = \frac{2^2}{3^1} = \frac{4}{3} = P4,$$ the perfect fourth,

$$\frac{9}{8}\cdot\frac{9}{8}\cdot\frac{256}{243}\cdot\frac{9}{8} = \frac{3^2}{2^3}\cdot\frac{3^2}{2^3}\cdot\frac{2^8}{3^5}\cdot\frac{3^2}{2^3} = \frac{2^8 3^6}{2^9 3^5} = \frac{3}{2} = P5,$$ the perfect fifth,

$$\frac{9}{8}\cdot\frac{9}{8}\cdot\frac{256}{243}\cdot\frac{9}{8}\cdot\frac{9}{8}\cdot\frac{9}{8}\cdot\frac{256}{243} = \frac{3^2}{2^3}\cdot\frac{3^2}{2^3}\cdot\frac{2^8}{3^5}\cdot\frac{3^2}{2^3}\cdot\frac{3^2}{2^3}\cdot\frac{3^2}{2^3}\cdot\frac{2^8}{3^5} = \frac{2^{16}3^{10}}{2^{15}3^{10}} = 2 = P8,$$ the perfect octave [major].

These Pythagorean Tones and semitones are not Whole steps and half steps.
Boethius considered the idea of "half" of a Pythagorean Tone, $\frac{9}{8}$, but realized that it was not possible because "half" of $\frac{9}{8}$ was no longer a ratio of numbers. Here's why.
Another way of putting this question is, What times itself equals $\frac{9}{8}$? Whatever that is, will be "half" of a Pythagorean Tone.

Using algebra we put it like this, $x^2 = \frac{9}{8}$.

And we solve it like this, $\sqrt{x^2} = \frac{\sqrt{9}}{\sqrt{8}} = \frac{3}{\sqrt{4\cdot 2}} = \frac{3}{2\sqrt{2}} = x$. That is a "half" of $\frac{9}{8}$, but that $\sqrt{2}$ is irrational and therefore not considered a number in premodern music theory.

$\frac{\sqrt{9}}{\sqrt{8}} = \frac{3}{2\sqrt{2}} \cong 1.060660172$ ... approximately. It's irrational. The decimal goes on forever without pattern.

$\frac{256}{243}$. It produces a long decimal (ca. 1.05349794238683) but the fraction is rational and exact.

$\frac{256}{243}$ is close to $\frac{\sqrt{9}}{\sqrt{8}}$, but not exactly the same.