QUADRIVIUM

Name_________________________

HW-4a [Homework 4a]

Answer the following questions:

i) What year and under what circumstances did Boethius die?

pp. 71-78

ii) In what order did Boethius put the 4 quadrivial disciplines?

iii) Explain the difference between magnitude and multitude.

iiii) Which does arithmetic deal with? Magnitude or multitude? [circle one]

v) Which does geometry deal wit? Magnitude or multitude? [circle one]

vi) Is the square root of two a magnitude or a multitude? [circle one]

vii) What are these? 3, 5, 9, 123, 44, 19 Magnitudes or a Multitudes? Comment if you want.

viii) What are these? π, 2, $\sqrt{2}$, 3/2 Magnitudes or a Multitudes? Comment if you want.

viiii) Finish this quote, "This, therefore, is the *quadrivium*...

x) Which is the primary and essential discipline, upon which all other disciplines rely?

xi) Briefly explain what an even number is.

xii) Briefly explain what an odd number is.

p. 79

Given the natural numbers [counting numbers]. 1, 2, 3, 4, 5, 6, **7**, 8, 9, 10, 11, 12, 13, 14, 15, 16, ...

If you select one of them... say 7... and you add up the numbers surrounding 7 on either side... 6 and 8, they add to twice 7. 6 + 8 = 14, which is twice 7. Furthermore, if you pick the numbers two positions away from 7 on either side, 5 and 9, they also add to 14, which is also twice 7. And three positions away and four positions away.

1) Pick another number in this series and see if this holds true. Write your results so that I can follow your method. Comment on these results. Write an algebraic generalization of this phenomena if you can.

pp. 80-81

2) Given the series of doubles up to 1024: 1, 2, 4, 8, 16, **32**, 64, 128, 256, 512, 1024.

The middle term of this series is 32. There are five terms to either side of 32. Analyze this series the same way Boethius does on p. 81 in the paragraph starting, "If, however, we take..." Write it up clearly so that the reader understands the pattern being described.

3) Does this property work on this truncated series? 8, 16, 32, **64**, 128, 256, 512

4) Do the properties from #2 work on this series of triples? 1, 3, 9, **27**, 81, 243, 729

5) Do these properties hole for this series based on even exponents? 1, 4, 16, **64**, 256, 1024, 4096

6) Do these properties hole for this series based on odd exponents? 2, 8, 32, **128**, 512, 2048, 8192

7) Do these properties hole for this series based on powers of 5? 1, 5, **25**, 125, 625

8) Do these properties hold for the series based on x^n, [where n = counting numbers and 0]? x^0, x^1, x^2, x^3, x^4, x^5, x^6

9) Can you make some general observations based on the evidence you have obtained by doing problems 2 through 8?

pp. 89-92

10) According to Boethius, is 2 a prime number?

11) How are prime numbers like atoms?

12) "Secondary and composite" numbers, according to Boethius, are odd numbers* that have prime factors. They are "secondary" because they are not "primary." Only prime numbers are "primary." That's why they are called "prime." A number is "composite" because it has more than one factor besides itself and Unity. For example, 9. It can be broken down into 3s. Or 21, ...it breaks down into 3 and 7... both primary numbers. Read over Ch.15 on pp. 91-91. Does 27 fit the definition as you understand it? Comment.

For Boethius a prime number is an odd natural number that has exactly two distinct natural number factors (or divisors), namely 1 and the number itself. No more.

Boethius: An odd natural number greater than 1 that is not prime is called a secondary and composite number. [Discussed above.]

How to find prime numbers?

One way is the Sieve of Eratosthenes (d. 194 BC). Boethius describes this on pp. 92-94. His description is beyond confusing. Below is another way to describe this process.

a. Start by crossing out all even numbers on the table of numbers below.

b. Go to the next number that is not crossed out. That would be 3. Is it a prime number? Can it be expressed as a product of two natural numbers less than 3? The only natural number candidates are 1 and 2. Can you multiply 1 x 1, 1 x 2, or 2 x 2 and get 3? No. Therefore 3 is prime. Its only factors are 1 and 3. Now cross out every 3rd number after 3. E.g. Cross out 6, 9, 12, 15, etc. I just count up by threes and scribble out the box...."4, 5, 6, scribble, 7, 8, 9, scribble, 10, 11, 12, scribble."

c. Now find the next number that is not crossed out. That should be a 5. [4 should be crossed out from step a.] Is 5 prime? Are there any factors of 5 that are natural numbers less than 5? If so, cross out every fifth number after 5. E.g. 10, 15, 20, etc. These are easy since they are in columns.

* I speculate that Boethius limits "secondary and composite numbers" to the odd numbers because all even numbers are obviously composite numbers since they always have 2 for a factor. For Boethius the category of "even" doesn't need to be included in "composite."

d. Go to then next number that is not crossed out, it should be a prime number. Repeat step c. Do this until you can't anymore.

What remains are the primes up to 100. This method will go as high as you are willing to do it. It's tedious, but effective.

~~1~~	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

13) Prime factorize the following. See Examples.

E.g.	$44 = 2 \cdot 2 \cdot 11$
E.g.	$39 = 3 \cdot 13$
1	93 =
2	133 =
3	49 =
4	1024 =
5	23 =
6	437 =
7	667 =
8	65 =

pp. 96-98: Superfluous, Diminished, and Perfect Numbers. Read over this section to figure out what these terms mean.

14) Multiple choice. S, D, or P. [Superfluous, Diminished, and Perfect]

E.g.	12 It's factors add:1+2+3+4+6 = 16 16 > 12, thus Superfluous.	S
E.g.	14 It's factors add: 1+2+7=10 10 < 14, thus Diminished	D
E.g.	28 It's factors add: 1+2+4+7+14 28 = 28, thus Perfect	P
a	24	
b	18	
c	8	

d	6	
e	26	
f	67	
g	496	
h	837	
i	56	

15) How are Superfluous and Diminished Numbers like monsters?

16) From pp. 103-105. Identify what type of ratio it is.

Definition 1: *Sesquialter* ratios are ones in which the numerator is $1\frac{1}{2}$ times the denominator.

E.g. $\frac{18}{12} = \frac{12(1.5)}{12}$. Thus this is *Sesquialter*.

Definition 2: *Sesquitertian* ratios are ones in which the numerator is $1\frac{1}{3}$ times the denominator.

E.g. $\frac{24}{18} = \frac{18\left(\frac{4}{3}\right)}{18} = \frac{\frac{72}{3}}{18} = \frac{24}{18}$. Thus it is *Sesquitertian*.

E.g.	$\frac{21}{14} = \frac{3}{2}$	*Sesquialter* [One and a half or 3/2]
E.g.	$\frac{16}{12} = \frac{4}{3}$	*Sesquitertia* [One and a third or 4/3]
a	$\frac{21}{14}$	

b	$\frac{28}{21}$	
c	$\frac{48}{36}$	
d	$\frac{15}{10}$	