26. The wheel of fortune. From Gregor Reisch, *Margarita Philosophica* (1503).

Assignment 14B:
-Annandale- Tuesday, May 9th

Quadrivium

- Read Chapters 5 and 6 from Yoko Ogawa's *The Housekeeper and the Professor*.
- Read Cardano's *Liber de Ludo Aleae*, pp. 185-196.
- Read this excerpt from D.P. Walker's classic book, *Spiritual and Demonic Magic*. (optional) This excerpt is about Ficino and his spirit-music. It is one of my all-time favorite selections ever. It puts physics onto astrology. Weird physics. Link on website. Password: "open". http://www.mifami.org/eLibrary/Walker-SDM-FicinoMusic-SAC.pdf
- Read pp. 92-93 in Cardano's *My Life*. (optional) This entire excerpt is awesome, but if you only read pp. 92-93 you'll get a taste of Cardano. He's a trip. http://www.mifami.org/eLibrary/Cardano-Book_of_My_Life-excerpt-PSR12.pdf
- Do the Mathy Homework (Problems 1-5).

To Self: Do summing to 100 and triangular numbers in class.

Ogawa:
Chapter 1: The housekeeper was born on Feb. 20.... 2 20... or 220. The professor's prize watch had the number 284 inscribed on it, "President's Prize 284."

220 and 284 are Amicable Numbers

220's divisors are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, which sum to 284.

284's divisors are 1, 2, 4, 71 and 142, which sum to 220.

The professor can do the divisors in his head. I can't. Here's how I do it....

-Start with the prime factorization of 220: $2^2 \cdot 5^1 \cdot 11^1 = 2 \cdot 2 \cdot 5 \cdot 11 = 220$.

-The exponents of the prime factors are 2, 1, and 1.
Add 1 to each to get, 3, 2, and 2 and then multiply these together: $3 \cdot 2 \cdot 2 = 12$.
That is how many divisors you are looking for: 12 divisors.

-Now calculate $\sqrt{220}$. It's about 14.83. Round your answer down to 14.

-Now just check all numbers from 1 to 14 as divisors of 220, as shown in the table to the right. All the divisors are there: 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, and 220.

Note: Throw out the highest number [220] when summing the divisors for finding amicable or perfect numbers.

divisors of 220	
1	220
2	110
3	n.a.
4	55
5	44
6	n.a.
7	n.a.
8	n.a.
9	n.a.
10	22
11	20
12	n.a.
13	n.a.
14	n.a.

The prime factorization of 284 is: $2^2 \cdot 71 = 284$.

And the divisors of 284 are... ...Your turn.

1) Add one to each exponent and multiply them together. That's the number of divisors.

2) Take the square root of 284 and round it down. This is the limit you need to check for.

3) Now just go through the numbers 1-? (from previous step). Remember you are only looking for a certain number of divisors (established in step 1). Once you find them all you don't need to keep going. Fill in the table to the right. It's easier than it looks. Lots of "n.a."

divisors of 284	
1	
2	142
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	

Mathy Homework: Turn this in on Thursday.

Finding Amicable Numbers:

In the 9th century Thābit ibn Qurra came up with a formula to find amicable numbers from a type of number now called a Thabit Number. Thabit Numbers are defined as $3(2^n)–1$, where n is a counting number (1, 2, 3, etc.)

E.g.1. If n = 1, then the Thabit number is $3(2^n) - 1 = 3(2^1) - 1 = 5.$

E.g.2. If n = 2, then the Thabit number is $3(2^n) - 1 = 3(2^2) - 1 = 11.$

1) On the table below, find the first 8 Thabit numbers, q, and put them in the proper column.
2) Then figure out the column called, "p." These are just Thabit Numbers shifted down one row.
3) Then figure out the column called, "r." (formula given on table)

If in any row p, q, and r are all prime numbers, you can calculate the corresponding amicable numbers from p, q, and r. Formulas for this calculation are given on the table.

E.g.3. In row $n = 1$, the value for p is not prime. It's a 2. Recall that by medieval standards 2 is not prime. and thus no amicable numbers will result from row 1.

However, in row $n = 2$, all three numbers are prime: 5, 11, and 71. So proceed to figure out the amicable numbers from that row. [Formulas in table.] See E.g.4.

E.g.4. Lower Amicable Number in row 2: $2^n(pq) = 2^2(5 \cdot 11) = 220$
Higher Amicable Number in row 2: $2^n r = 2^2(71) = 284$

4) Determine if all three numbers (p, q, r) are prime in any given row. (I suggest you bring up a list of prime numbers on the web.) If you find an all-prime row, then figure out the corresponding Amicable Numbers. If all three are <u>not</u> prime, then there are no corresponding amicable numbers and there is no need to calculate the last two rows. There are 2 more pairs in this table.

n	p $p = 3(2^{n-1}) - 1$	Thabit Number, q $q = 3(2^n) - 1$	r $r = 9(2^{2n-1}) - 1$	Lower Amicable Number $2^n(pq)$	Higher Amicable Number $2^n r$
1	**2**	**5**	**17**		
2	**5**	**11**	**71**	**220**	**284**
3	11				
4					
5			4607		
6		191			
7	191				
8			294,911		

Here are two Amicable Numbers re-discovered by Fermat in the 17th century: 17,296 and 18,416.
Here's how they are amicable...

The divisors of 17296 add up to...

1+2+4+8+16+23+46+47+92+94+184+188+368+376+752+1081+2162+4324+8648 = **18,416**

And the divisors of 18,416 add up to...

1+2+4+8+16+1151+2302+4604+9208 = **17,296**

Thabit's technique didn't find all of the Amicable Numbers. Not by a long shot.

5) Below are two pairs of numbers. Which pair is amicable?

Prove it by finding all the divisors and adding them up.

[Hint: To find the divisors, use the technique from pp. 1 and 2.]

1210 and 1184 or 2620 and 2924