

Base-60 and Base-10

Our Decimal Number System

| Modern Decimal Number | Decimal Parts $10^3 + 10^2 + 10^1 + 10^0$ | Decimal equivalent |
|-----------------------|--|--------------------|
| 1523 | $1(1000) + 5(100) + 2(10) + 3(1)$ | $1000+500+20+3$ |
| 4085 | $4(1000) + 0(100) + 8(10) + 5(1)$ | $4000+0+80+5$ |

The sexagesimal system is based on the number 60 instead of 10. We still use it for hours:minutes:seconds. We count to 59 seconds and the next second gives us 1 minute. We count to 59 minutes and the next minute gives us 1 hour. Using one method of notation for sexagesimal we might write 1 hour, 25 minutes, and 15 seconds as
 $1:25:15$. (Like a digital clock.)

The next increment above an hour is unfortunately not base-60, it's base-24, otherwise known as a day. And then days are organized into base-7 weeks, then base-365.25 years, and only then does it settle into good old base-10: decades, centuries, millennia. So our time measurement system is a bit of a disaster, mixing number systems all over the place. You'll notice that I didn't even mention months, which are not in any consistent number system.... base-30, -31, -28, -29? What a mess!
 No wonder little kids have so much trouble mastering this stuff.

But let's get back to base-60.

1 hour, 25 minutes, and 15 seconds can be written, $1:25:15$.

Sacrobosco, however, is going to write this in boxes, like this:

| | | | |
|---|---|----|----|
| 0 | 1 | 25 | 15 |
|---|---|----|----|

You'll notice that none of the numbers the Alfonsine table excerpt is above 59. That's because the next number after 59 is 60... which is 1 in the next slot to the left. This is just like how we tell time. We don't say 7:60 p.m. We say 8:00 p.m.

Trivia: Minutes:Seconds.

Our word, "minute," comes from Latin, *pars minute prima*, "the first minute (*mine-oot**) part."

Fractional sexagesimal numbers are divisions based on sixtieths: $\frac{1}{60^1} + \frac{1}{60^2}$.

Our minute is the "first minute (*mine-oot*) part," $\frac{1}{60^1}$ (*pars **minute** prima*)

and our second is literally the second minute part, $\frac{1}{60^2}$. (*pars minute **secunda***)

Mine-oot part of what? *Mine-oot* part of an hour.

* 'mine-oot' is my way of phonetically spelling "minute" so that you pronounce it differently... so that it means tiny.

| Modern notation of Sexagesimal Number | Sacrobosco's Notation of a Sexagesimal Number | | | | Sexagesimal Parts $60^3 + 60^2 + 60^1 + 60^0$ | Decimal equivalent |
|---------------------------------------|---|----------|----------|----------|--|--------------------|
| | 4^{st} | 3^{rd} | 2^{nd} | 1^{st} | | |
| $60^3:60^2:60^1:60^0$ | | | | | | |
| 0:0:0:59 | 0 | 0 | 0 | 59 | $0(216,000) + 0(3600) + 0(60) + 59(1)$ | 59 |
| 0:0:1:0 | 0 | 0 | 1 | 0 | $0(216,000) + 0(3600) + 1(60) + 0(1)$ | 60 |
| 0:0:59:59 | 0 | 0 | 59 | 0 | $0(216,000) + 0(3600) + 59(60) + 59(1)$ | 3599 |
| 0:1:0:0 | 0 | 1 | 0 | 0 | $0(216,000) + 1(3600) + 0(60) + 0(1)$ | 3600 |
| 4:16 | 0 | 0 | 4 | 16 | $0(216,000) + 0(3600) + 4(60) + 16(1)$ | 256 |
| 1:2:4:7 | 1 | 2 | 4 | 7 | $1(216,000) + 2(3600) + 4(60) + 7(1)$ | 223,447 |
| 3:0:31:58 | 3 | 0 | 31 | 58 | $3(216,000) + 0(3600) + 31(60) + 58(1)$ | 649,918 |