



SOME MATHEMATICS
PERTAINING TO REGULAR POLYGONS
INSCRIBED IN A CIRCLE

A Few Mathematical Properties of an Equilateral Triangle Inscribed in a Circle	Example: equilateral triangle (see above)
1) All interior angles of a triangle add up to 180° .	$\theta = 60^\circ$
2) Interior angles (θ) are bisected by radii drawn from the center of the circle to a vertex.	$\varphi = \frac{\theta}{2} = \frac{180^\circ - \gamma}{2} = 30^\circ$
3) Angles between rays γ which extend to vertices measure $360^\circ/3$. [Note: $\gamma = \psi$]	$\gamma = 120^\circ$
4) Angles formed by drawing a radius to the middle point of any side of the triangle will be perpendicular where it meets the side and will measure $360^\circ/6$ in relation to radii from #3.	$\lambda = \frac{\gamma}{2} = 60^\circ$
5) Triangles formed from the center with sides extending to the tips of the polygons are isosceles.	See shaded region.

Determine the analogous properties of a few additional regular polygons in a circle. Refer to diagrams.

Mathematical Properties of a Square in a Circle.	Mathematical Properties of a Pentagon in a circle.	Mathematical Properties of a Hexagon in a circle.
1) $\theta =$	1) $\theta =$	1) $\theta =$
2) $\varphi =$	2) $\varphi =$	2) $\varphi =$
3) $\gamma =$	3) $\gamma =$	3) $\gamma =$
4) $\lambda =$	4) $\lambda =$	4) $\lambda =$
5) Shade in an isosceles triangle.	5) Shade in an isosceles triangle.	5) Shade in an isosceles triangle.
6) $\omega =$ [Duh]	6) $\omega =$	6) $\omega =$

Let n be the number of sides of the regular polygon. E.g. For a square, $n = 4$. For an equilateral triangle, $n = 3$. And let the radius of the circle be one, $r = 1$.

Your homework to be turned in by next Wednesday:

1) Using λ (or ω or φ) and \sin (or \cos or \tan), and x (see diagrams above), figure out the **general formula** for the **perimeter of an inscribed regular polygon**? This will be a formula by which one could input any number of sides, n , and output a perimeter. You must use λ (or ω or φ) and \sin (or \cos or \tan), and x as your terms. In other words, you'll end up with a formula that has an angle, a trig. function, and x . **Write this up so that I can follow your reasoning.** This will probably require the drawing of diagrams and some prose.

Hint: Look at the patterns in the data from the table on p1. Generalize these patterns into algebraic expressions. Also, the video class shows how this can be done.

2) Make a spread sheet that uses your perimeter formula from above. Make it so that you can input the number of sides, n , and it will output the perimeter. How many sides do you need in order to approximate π to 5 decimal places? ...meaning that the polygon's perimeter is accurate up to 3.14159. What is the lowest n resulting in 3.14159 (when rounded appropriately). Here's a picture of one I made that you could use as a model. I didn't include any spreadsheet formulae this time, so you'll have to figure out how to make this work. [Hint: Excel doesn't like degrees. That's why there is a section for radians.] Take a picture of it with your value for n to make 3.14159 and send that to me for this part of the homework.

	A	B	C	D	E
1	Poly to π				
2					
3		Inputs			
4	r =	1			
5	n =	7			
6					
7		Outputs in degrees	Output in rad		
8	$\gamma = 360/n =$	51.42857143	0.897597901		
9	$\lambda = \gamma/2 =$	25.71428571	0.448798951		
10					
11	$x = r \sin \lambda =$	0.433883739			
12	$P(n) = 2xn =$	6.074372348			
13			π for comparison		
14	$P/2r =$	3.037186174	3.141592654		