Sine, Cosine, and Tangent

$$sin = \frac{opposite}{hypotenuse} \qquad cos = \frac{adjacent}{hypotenuse} \qquad tan = \frac{sin}{cos} = \frac{\frac{opposite}{hypotenuse}}{\frac{adjacent}{hypotenuse}} = \frac{opposite}{adjacent}$$

csc, *sec*, and *cot* are just the reciprocal functions of *sin*, *cos*, and *tan* respectively: *1/sin, 1/cos, 1/tan.*

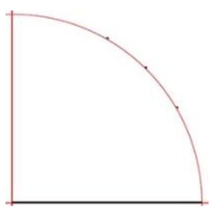
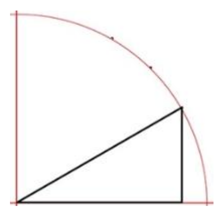
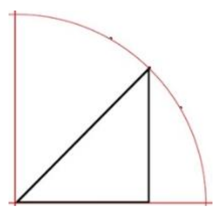
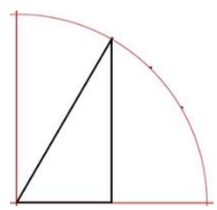
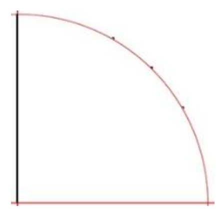
A way to remember the sine of commonly encountered angles:

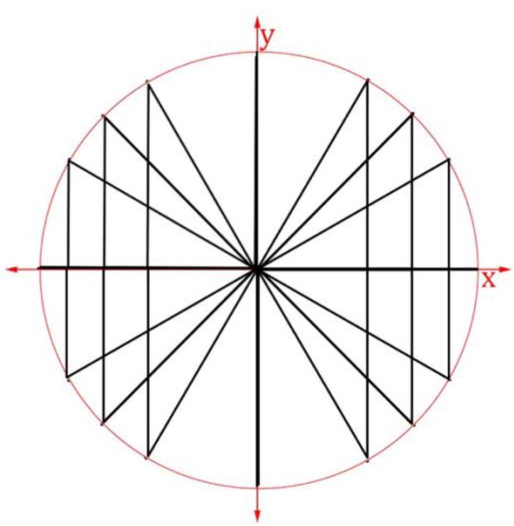
$\frac{\sqrt{0}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}$ are the sine of 0º, 30º, 45º, 60º, 90º $or\ in\ radians,\ 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

The cosine is just reversed.

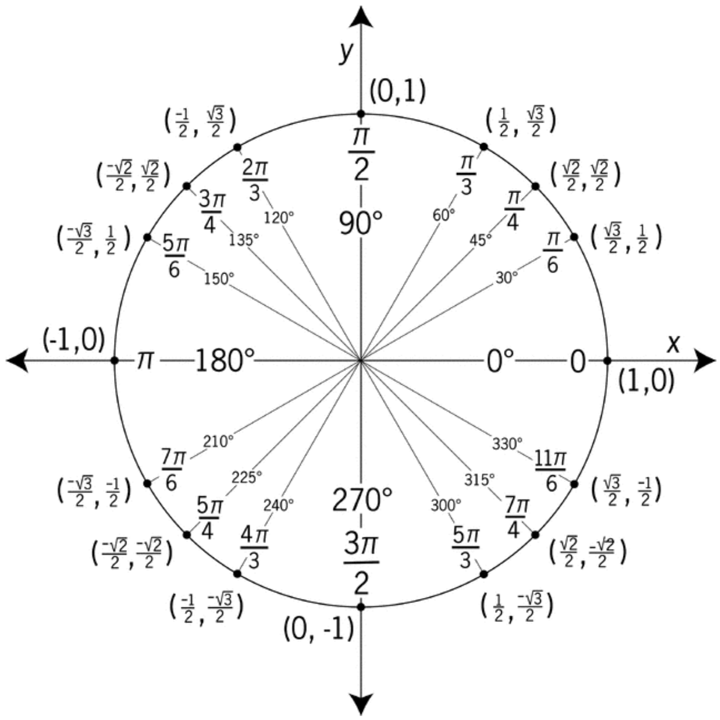
$\frac{\sqrt{4}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{0}}{2}$ for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$.

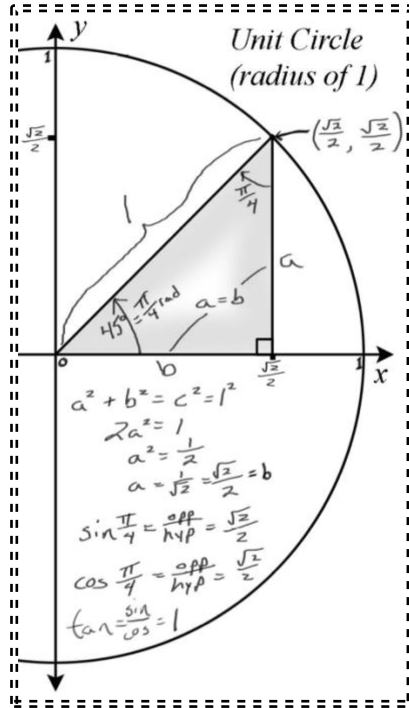
First Quadrant Landmark Triangles

					
radians	$0\ rad$	$\pi/6 \approx 0.52\ rad$	$\pi/4 \approx 0.79\ rad$	$\pi/3 \approx 1.05\ rad$	$\pi/2 \approx 1.57\ rad$
degrees	0º	30º	45º	60º	90º
sine	0	$\frac{1}{2} = 0.5$	$\frac{\sqrt{2}}{2} \approx 0.707$	$\frac{\sqrt{3}}{2} \approx 0.866$	$1 = 1$
cosine	$1 = 1$	$\frac{\sqrt{3}}{2} \approx 0.866$	$\frac{\sqrt{2}}{2} \approx 0.707$	$\frac{1}{2} = 0.5$	0



If you keep going around, you get all these triangles, which have the same dimensions as the triangles detailed in the first quadrant. [See table above.]





The chart on the left shows each landmark *θ in terms of cosθ* and *sinθ* ...as a point, (*x*, *y*) = (*cosθ*, *sinθ*), in terms of degrees and radians on a unit circle (*r* = 1). The diagram on the right is a detail of the right half of a **unit circle** and it focuses on π/4 or 45º. It shows how the Pythagorean Theorem is involved in the determination of sine and cosine. The ordered pair is (cos, sin), which is slightly annoying because the order seems backwards. This is something you'll just have to remember or derive.

Trigo-no-metry (triangle measurement) is the Pythagorean Theorem turned into algebra.
The "x-axis" measurements are cosine because they measure the adjacent leg of the right triangle.
The "y-axis" measures sine, because the sine involves the opposite leg of the right triangle.

The Unit Circle. $(cos\theta, sin\theta)$

(The "Unit Circle" has a radius of 1... unit... as in unicycle, unison, universe, etc.)
The beauty of the unit circle is that the hypotenuse is equal to 1 for each and every right triangle formed in/on the unit circle. For sine and cosine this is really great since they both have the hypotenuse in the denominator. A denominator of one doesn't get any easier.

Homework, not to be turned in:

1) Read over Chapter 18, paying special attention to Example 18.4.2 on p248.

2) Do the following problems at the end of Ch18: 18.1, 18.3, and 18.5.

3) Do the following worksheet and turn in a write-up. Explained below.

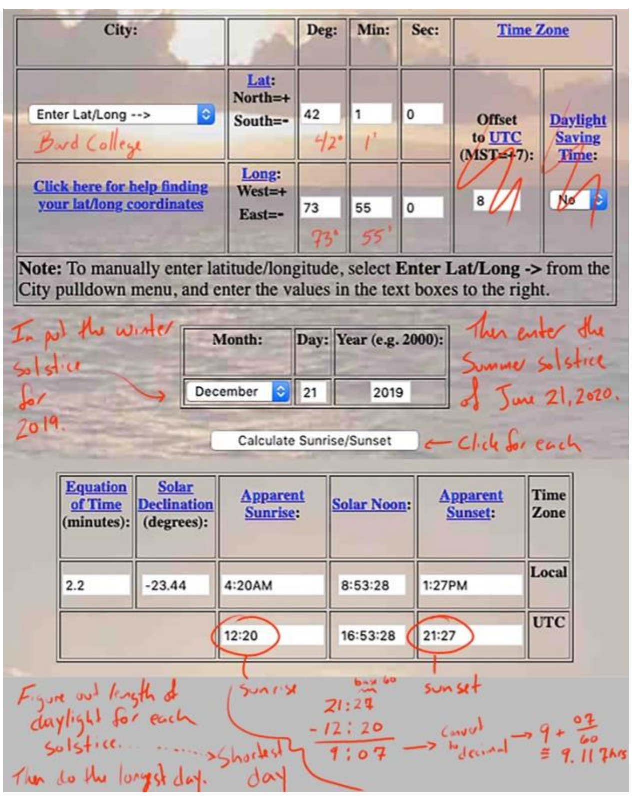
See next page...

The "Turn-In" Homework

Since we are not having a final exam, let's make this assignment serve as a summary of the trigonometry unit. It's a larger homework assignment, but it is not due until Monday, 5/25. There will still be another shorter assignment or maybe two more, but consider this one to be the big one... the one to focus on. Write it up very nicely. Definitely put in graphs. Use all your skills to make this an awesome presentation. Feel free to get artsy on it. Go nuts. I'll grade it as much on its artistic merit and presentation as on its mathematical righteousness. Use your own judgment for how much to turn in and which aspects of the project warrant the most attention. Feel free to present the requested information in any format you want. No need to use this PDF as a template. However, include all of the requested material, especially the final equation for your location (Eq. 1). If you have trouble coming up with this equation, contact me and I'll help you out. Don't wait until the last second!

Worksheet for determining the length of a day at your location.

a) What is the latitude and longitude of your current location in degrees and minutes. [Latitude (& Longitude) in GoogleMaps] I looked up Bard and for the latitude I got: 42.02ºN. That's 42º plus 0.02(60 sec) ≈ 42º 1'N. And for the longitude I got 73.91º which translates to 73º55'.



b) Now choose your city or plug your coordinates into this site: NOAA: Sunrise/Sunset Calculator. If your city is not in the list, choose the "Enter Lat/Long" option at the top of the drop-down menu and enter your latitude and longitude from part a. Don't bother with "Time Zone" stuff. Leave that alone.

c) Enter December 21, 2019 and hit, "Calculate Sunrise/Sunset" and jot down both the "Apparent Sunrise" and the "Apparent Sunset" times. This is the shortest day of the year (in terms of sunlight in the northern hemisphere). See picture below. Then change the date to June 21, 2020 and get the Sunrise and Sunset times from that day... the longest day of the year.

c) Calculate the length of the shortest day from a few months ago (winter solstice, Dec. 21, 2019) and then calculate the upcoming longest day (summer solstice, June 21, 2020). Convert results to decimal.

For example... For Bard I ended up with...

Shortest day: 9:07 (9 hours and 7 minutes of daylight) 9:07 = 9.12 hrs.

Longest day: 15:15 (15 hours and 15 minutes of daylight) 15:15 = 15.25 hrs.

Find the midpoint between 9.12 and 15.25. Meaning... add together and divide by two.

$$9.12 + 15.25 = 24.37 \quad \rightarrow \quad Average\ is\ 12.185\ hours.$$

This makes a certain amount of sense. The average should be 12 hours... right? The average days' worth of daylight should be about 12 hours. The reason it is not exactly 12 is because of various astronomical issues, the main issue being our elliptical orbit around the sun. This complicates the situation, but not enough to invalidate our estimate.

If the average is 12.185, the variation above and below the average is $\pm(longest - average)\ or\ \pm(15.25 - 12.185)$, which should be the same as $\pm(\ average - shortest)\ or\ \pm(12.185 - 9.12)$, ... the answer for a Bard latitude being ± 3.065. At Bard, the variation above and below the average amount of daylight is $\pm 3.065\ hours$. The longest day has 15.25 hours and the shortest day has 9.12 hours and the average day is 12.185 hours long.

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d) Put all of the data for <u>your location</u> here:

					k	A
City, Country	Latitude in Decimal	Longitude in Decimal	Length of Shortest Day in decimal	Length of Longest Day in decimal	Average Day in decimal	Variation above and below average
						$(\pm)$

Here is the rest of the relevant data not subject to local variation.

Length of a year in days	Approximate length of a season*	Difference between the winter solstice and the New Year. Dec. 21 and Jan. 1.
365.25 days	$\frac{365.25}{4} = 91.3125\ days$	10 days

e) We now have all the necessary numbers to plug into an equation that models the amount of daylight hours on a given day of the year at your location. Here's the equation.

$$d(t) = A sin\left\{\left(\frac{2\pi}{365.25}\right)[t - (91.3125 - 10)]\right\} + k \qquad \textbf{Equation 1}$$

The constants A and k are on the table above.

[You don't need to put the "$\pm$" in front of A in the equation.]

The equation for Bard looks like this:

$$d(t)_{Bard} = (3.065) sin\left\{\left(\frac{2\pi}{365.25}\right)[t - (91.3125 - 10)]\right\} + 12.185$$

* Once again, due to the elliptical nature of earth's orbit, the length of a season is not exactly one quarter of a year, but it's close enough.

Write the equation for <u>your location</u> here:

To use this formula, enter the day number, *t*, and it will output the hours of daylight on that day. The day number is annoying. You have to count them up starting on January 1st.

For example, how many hours of daylight will there be at Bard on May 20th?

May 20 is .... Jan. + Feb. + March + April + 20 = 31 + 29 + 31 + 30 + 20 = the **141st day**

t = 141

I input 141 into the equation using the Bard example equation:

$$d(141)_{Bard} = (3.065)sin\left\{\left(\frac{2\pi}{365.25}\right)[141 - (91.3125 - 10)]\right\} + 12.185$$

The output for 141:

$$d(141) = 14.81\ hours\ of\ daylight\ on\ May\ 20, 2020.$$

14 hours, 48 minutes, 36 seconds = 14:48:36

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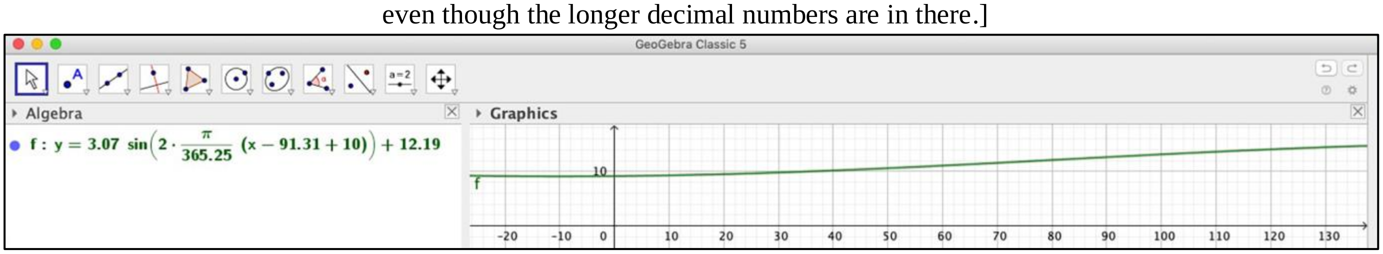
f) Now put <u>your equation</u> into either Desmos or Geogebra or similar (with *d(t) = y* and *t = x*).

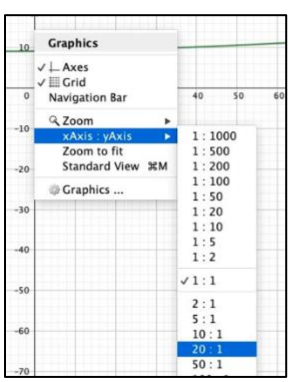
Here's the Bard equation I'm using as an example in Geogebra.

Getting all the parentheses right is the hardest part of typing in this equation.

[For some reason Geogebra rounded all decimals to two places in the display of the equation, even though the longer decimal numbers are in there.]

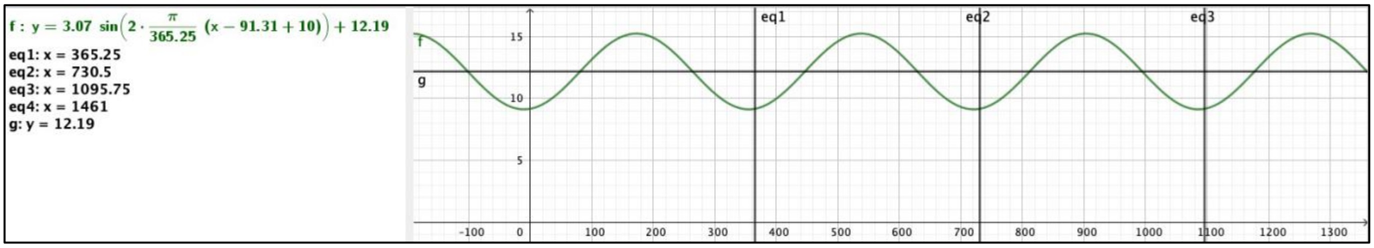


You <u>can't</u> clearly see what's going on in this graph without re-scaling the axes.



In Geogebra, right-click on one of the axes and you get a dialogue box as shown here (left). I found that 20:1 worked out well, but feel free to try others.

A bit more fiddling with settings and adding a vertical line for every year and a horizontal line on the average and I got this graph:



You should now have a working graph.

If you don't, email me with your location and we'll figure it out.

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g) Answer the following questions. Answer them completely and thoughtfully. Answer them as if you were explaining this equation and graph to your little brother or sister who has just started studying algebra in school. Explain how the equation affects the graph... translations, shifts, stretching, altering periodicity... etc. This is the most important part of this assignment. Feel free to refer to your specific equation in addition to the general equation, Equation 1.

$$d(t) = A\sin\left\{\left(\frac{2\pi}{365.25}\right)[t-(91.3125-10)]\right\} + k$$ -Equation 1 (general equation)

i) What does *A* do? Why is it there? What happens if you remove it?

ii) What does *k* do? Why is it there? What happens if you remove it?

iii) What does the $\frac{2\pi}{365.25}$ do? Why is it there? What happens if you make it 1?

iv) Why "91.3125 – 10"? What does it do? What happens if you make it 0?

v) What does the "–10" do? What happens if you remove it? How does that change how you read the graph?

vi) Come up with numbers for *A* and *k* that make sense for a location <u>exactly on the equator</u>. Run this equation through a graphing program and see if it does what you hoped it would do. Describe your thought process as you do this. Feel free to describe the bad ideas along with the good ones.

vii) Come up with numbers for *A* and *k* that make sense for the North Pole. Run this equation through a graphing program and see if it does what you hoped it would do. Describe your thought process as you do this. Feel free to describe the bad ideas along with the good ones.

vii) Rewrite your equation in terms of cosine, instead of sine. You'll know if you did it right if it graphs exactly the same. Comment on how this works.

viii) Look up the Latin roots of *solstice* and *equinox* and say something about them as they relate to your graph and equation.

That's it. Feel free to add anything that you think will add to this write-up.

Length of Day links:
Hours of light per day based on latitude/longitude formula

https://en.wikipedia.org/wiki/Sunrise_equation

https://en.wikipedia.org/wiki/Declination

Older NOAA Sunrise/Sunset Calculator: https://www.esrl.noaa.gov/gmd/grad/solcalc/sunrise.html

Updated NOAA Sunrise/Sunset Calculator: https://www.esrl.noaa.gov/gmd/grad/solcalc/

http://www.jgiesen.de/astro/solarday.htm- calculator