7.7 Exercises

Problem 7.1. Write the following quadratic functions in vertex form, find the vertex, the axis of symmetry and sketch a rough graph.

(a) $f(x) = 2x^2 - 16x + 41$.

(b) $f(x) = 3x^2 - 15x - 77$.

(c) $f(x) = x^2 - \frac{3}{7}x + 13$.

(d) $f(x) = 2x^2$.

(e) $f(x) = \frac{1}{100}x^2$.

Answer 7.1 (a) $2(x-4)^2 + 9$, Vertex: (4, 9), Axis: x = 4. (b) $3(x - 5/2)^2 - 383/4$, Vertex: (5/2, -383/4), Axis: x = 5/2. (c) $(x - 3/14)^2 + 2539/196$, Vertex: (3/14, 2539/196), Axis: x = 3/14. (d)$2(x-0)^2 - 0$, Vertex: (0, 0), Axis: x = 0. (e) $(1/100)(x-0)^2 - 0$, Vertex: (0, 0), Axis: x = 0.

Problem 7.2. In each case, find a quadratic function whose graph passes through the given points:

(a) $(0,0)$, $(1,1)$ and $(3,-1)$.

(b) $(-1,1)$, $(1,-2)$ and $(3,4)$.

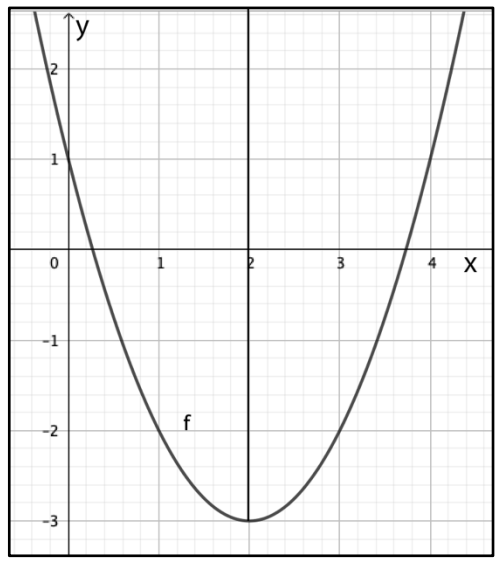
(c) $(2,1)$, $(3,2)$ and $(5,1)$.

(d) $(0,1)$, $(1,1)$ and $(1,3)$.

Answer 7.2 (a) $y = -\frac{2}{3}x^2 + \frac{5}{3}x$. (b) $y = 1.125x^2 - 1.5x - 1.625$. (c) $y = -0.5x^2 + 3.5x - 4$. (d) No solution.

Notes on 7.2: Figure out the quadratic formed with three points.
Instead of starting with 3 points, let's start with a known quadratic and reverse engineer it.

$$f(x) = y = (x-2)^2 - 3$$



The axis of symmetry is simply the x-coordinate of the vertex: (x = 2).

If we expand the vertex form we get...
$$y = x^2 - 4x + 1$$

Set y equal to zero,
run it through the quadratic equation,
and we find the x-coordinates at y = 0.

$$\left(2+\sqrt{3}, 0\right) \; and \; \left(2-\sqrt{3}, 0\right)$$

We now have several points to choose from:

$$\left(2+\sqrt{3}, 0\right) \quad \left(2-\sqrt{3}, 0\right) \quad (0,1) \quad (2,-3) \quad (4,1)$$

For the sake of demonstration, let's pick three different points and see if the same quadratic results. I'll pick the easy points to make the calculations easier:
$(0,1) \;\; (4,1) \;\; (2,-3)$

The chosen points	$y_n = ax_n^2 + bx_n + c$	Solve for c, b, and a.	Solve more.
$Point\ 1$: $(0,1)$	$y_1 = 1 = a0^2 + b0 + c$	$1 = c$	$c = 1$
$Point\ 2$: $(4,1)$	$y_2 = 1 = a4^2 + b4 + c$	$1 = a4^2 + b4 + 1$	$b = -4a$
$Point\ 3$: $(2,-3)$	$y_3 = -3 = a2^2 + b2 + c$	$-3 = 4a + 2(-4a) + 1$	$a = 1$

Solve for b with a = 1 and you get b = –4.

We now have values for a, b, and c: 1, –4, and 1 respectively.

Put these into the standard quadratic form: $y = ax^2 + bx + c$.

$$\boldsymbol{y = x^2 - 4x + 1} \quad \boldsymbol{Ta\ Da}!$$

Three points, three equations, and the quadratic is discovered.

Complete the square [calculations not shown] to get the vertex form and we go full circle:

$$y = (x-2)^2 - 3$$