BLC190-ATF [mifami.org] WORKSHEET 9 SPECIAL TOPICS

1. Break up into pairs or triplets. Using the **inclinometer** on a smart phone (the iPhone compass has this feature) and a **measured pace,** figure out the exterior height (or interior if it is raining) of RKC. Practice pacing consistently and then measure off 5 paces and divide by 5. Do this a couple of times.

You'll need to collect three pieces of information:

1) Angle above horizontal ...from your eye to the thing.	
2) Distance in measured paces to a point directly below the thing.	
3) How high your eye is from the ground.	

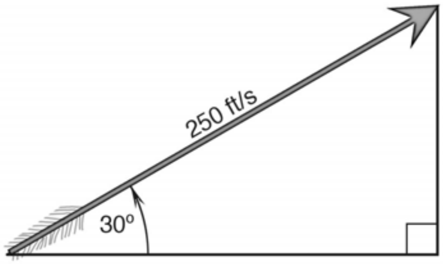
Then do the math...

2. Figure out the height of the balcony floor of RKC. I'll put a yellow flag to identify where to measure.

3. Figure out the height of one of you... or all of you.

4. If the North Star is visible, use it to determine the height of something like a tree or a building or a person.

5a. Odysseus' Arrow: Using the kinematic equation (below), determine the time it will take for this arrow to reach its apogee.

	$v_f = v_i + gt$ v_i is the initial vertical velocity (find using trig.), v_f is the final vertical velocity at peak altitude, 0 ft/sec, g is gravity measured to be *-32ft/s²* t is the time to peak altitude (apogee)

5b. Now determine the maximum height (apogee) using quadratic function that describes its entire vertical trajectory using the kinematic equation...

$$y_f = v_i t + \tfrac{1}{2}(gt^2)$$

where y_f is the altitude.
v_i is the vertical component of the initial velocity
t is the time to get to the apogee

5c. Now use the equation from 5b and write it as the function, *y(t)*, and graph it with zero points and vertex location (which you already know by when the vertical velocity is 0). You might not have time to do this... so on the next 2 pages is a summary of how it is all done (except for the vertex form).

Odysseus, disguised as a beggar, strings a bow which none of the suitors could. Longbows are typically stored unstrung. To use a long bow you need to string it. But a really powerful longbow is very hard to string.

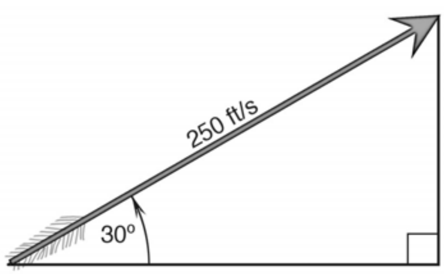
You have to bend it. But Odysseus is a demigod. So let's say the bow in question is a 200lb draw. The highest draws on modern long bows are about 70lb. Supposedly the maximum draw on English long bows several hundred years ago was about 100lbs.

A 200lb bow will launch an arrow with an initial velocity of 250 ft/sec.

If Odysseus looses an arrow with this bow at a 30º angle, how high and far will this arrow go.

For simplicity let's make a few assumptions:

- -No air resistance. He's shooting in a vacuum.
- -He is shooting on a very large flat field.
- -The arrow starts at an altitude of 0 feet and lands in the ground at 0 feet. We are disregarding how tall Odysseus is.



In order to do this problem we need to figure out how long the arrow will be in flight. The primary force limiting the arrow's travel is gravity (since we are ignoring air resistance). Gravity is what will push the arrow back down and into the ground. Gravity only operates vertically. Gravity has no effect on the horizontal progress of the arrow. So it is the gravity that determines how long the arrow will be aloft. To determine this we use one of the kinematic equations:

$$v_f = v_i + gt,$$

where v_f is the final velocity, v_i is the initial velocity, g is gravity, and t is time.

The problem is, our initial velocity is at a 30º angle. Since gravity only affects the vertical velocity, we need to extract just the vertical component of this velocity. Enter trigonometry.

$$sin30° = \frac{y}{250\ ft/s}$$

$$\boldsymbol{y = 125\ ft/s} \cong 85.2\ miles/hour\ \cong 137.1\ km/hr$$

This is the purely vertical component of the initial velocity. (It's about the speed of a major league change-up, ... thrown straight up.) Another thing to realize is that the vertical velocity will be zero at its maximum height. That's where the arrow stops going up, and starts coming down.

The kinematic equation (from physics) most useful for this vertical component is ...

$$v_f = v_i + gt$$

where v_f is the final vertical velocity at the peak altitude, 0 ft/sec, v_i is the initial vertical velocity, 125 ft/sec and gravity is measured to be $-32ft/s^2$

$v_i = 125\ ft/s$ $\quad$ $v_f = 0\ ft/s$ $\quad$ $g = -32ft/s^2$

$$v_f = v_i + gt$$

$$0\ ft/s = 125ft/s - 32ft/s^2(t)$$

$$0\ = 125 - 32t$$

Solve for t.

$$\boldsymbol{t \cong 3.9\ seconds}$$

That's the amount of time it takes to reach its apogee (highest point).

How high was it?

Use another of the kinematic equations:

$$\Delta y = y_f - y_i = v_i t + \tfrac{1}{2}(gt^2).$$

We know the initial y altitude, y_i, since it was set up as zero. The arrow is shot from ground zero. The only one we don't know is y_f, the final altitude. So, just plug in everything we know...

$$y_f - y_i = v_i t + \tfrac{1}{2}(gt^2)$$

$$y_f - 0ft = 125ft/s(3.9s) + 0.5(-32ft/s^2)(3.9s)^2$$

$$y_f = 487.5ft - 243.36ft = 244.14ft$$

That's pretty high!

We can also graph this vertical trajectory as a function of time, $y(t)$, and not just stop at the apogee.

$$y(t) = 125ft/s(t) + \tfrac{1}{2}(-32ft/s^2)(t^2)$$

$$y(t) = 125t - 16t^2$$

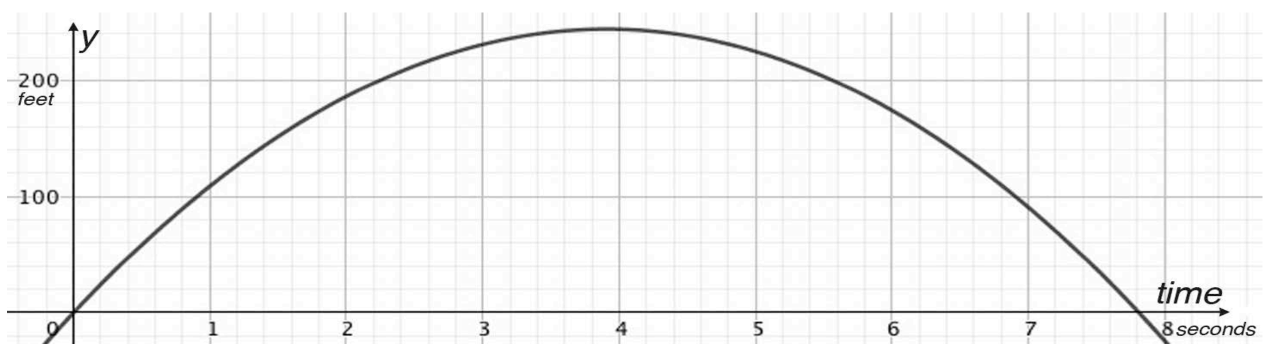
You can see where y = 0 by factoring the t, out.

$$y(t) = t\left[125\frac{ft}{s} + 0.5\left(32\frac{ft}{s^2}\right)t\right]$$

$$y(t) = t[125 + 16t]$$

The altitude of the arrow, y, is 0 when t is 0, at the beginning of the shot,

and when t is 7.8125 seconds, when the arrow strikes the ground at the end of the shot.



As you can see, at 3.9 seconds the arrow is at its maximum altitude of 244.14 ft.

The horizontal distance can be found by simply figuring out the horizontal component of the arrow's velocity and multiplying it by how long is in the air..... 7.8125 seconds.

Addendum: To officially find the vertex, use the vertex form: (completing the square)

$$y = -16t^2 + 125t$$

$$\frac{y}{-16} \cong t^2 - 7.81t$$

$$\frac{y}{-16} \cong t^2 - 7.81t + 15.25 - 15.25$$

$$\frac{y}{-16} \cong (t - 3.9)^2 - 15.25$$

$$y \cong -16(t - 3.9)^2 + 244$$

Vertex is at approximately (3.9, 244), which we already knew.