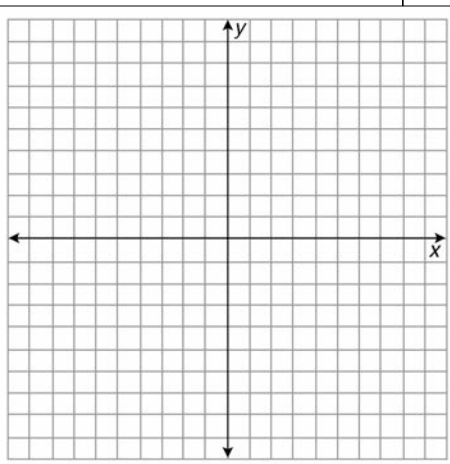
Solutions to

BLC190-ATF [mifami.org] HOMEWORK 8 SPECIAL TOPICS

Find the vertices of the following quadratic equations. Use either method described in the Class 8 Handout. And also identify if they are maximums or minimums. Only graph 1 and 2 on the provided graph. Use scrap paper when needed.

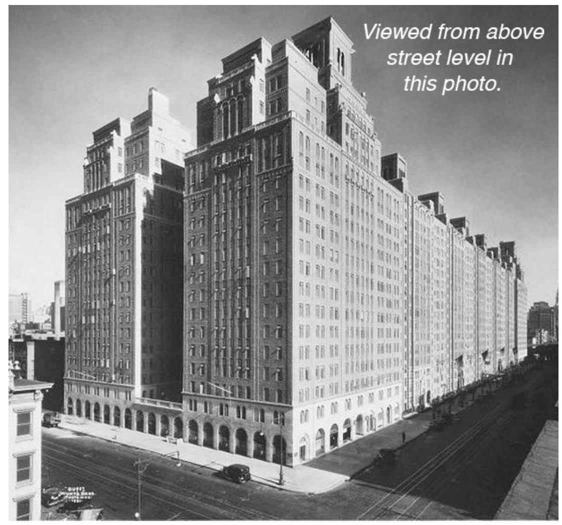
Problems	Location of Vertex	Max or Min?
1. $y = -3x^2 + 24x - 41$	$y/(-3) = x^2 - 8x + 41/3$ $h = -8/2$ $h^2 = 16$ $y/(-3) = x^2 - 8x + 16 - 16 + 41/3 = (x - 4)^2 - 7/3$ $y = -3(x - 4)^2 + 7$ The vertex is here: (4, 7)	max
2. $y = 5x^2 + 10x + 2$	$\frac{y}{5} = x^2 + 2x + \frac{2}{5}$ $\{ h = \frac{2}{2}$ $h^2 = 1$ $= x^2 + 2x + 1 - 1 + \frac{2}{5}$ $y = 5(x+1)^2 - 3$ vertex: $(-1, -3)$ $5(x + 1)^2 - 3$ (-1, -3)	min
3. $y = 12x^2 - 48x + 32$	$\frac{y}{12} = x^2 - 4x + \frac{32}{12} + 4 - 4$ $\{ h = \frac{-4}{2}$ $h^2 = 4$ $y = 12(x^2 - 2)^2 - 16$ vertex: $(2, -16)$ (2, -16)	min
4. $y = -6x^2 + 6x - 0.5$	$\frac{y}{-6} = x^2 - x + \frac{0.5}{6} + \frac{1}{4} - \frac{1}{4}$ $\{ h = \frac{-1}{2}$ $h^2 = \frac{1}{4}$ $y = -6(x - \frac{1}{2})^2 + 1$ vertex: $(\frac{1}{2}, 1)$ $-6(x - 0.5)^2 + 1$ (0.5, 1)	max
5. $y = -0.5(32)x^2 + 125x + 0$	$\frac{y}{-16} = x^2 - \frac{125}{16}x$ $h = \frac{-125}{32}$ $h^2 = (\frac{125}{32})^2$ $= x^2 - \frac{125}{16}x + (\frac{125}{32})^2 - (\frac{125}{32})^2$ $y = -16(x - \frac{125}{32})^2 + 16(\frac{125}{32})^2$ $y \cong -16(x - \frac{125}{32})^2 + 244$ approx. vertex: $(3.9, 244.1)$	max



6. How tall is the tree if you are one football field away from it and the angle you measure from your feet to the top of the tree is 3.4º above horizontal? Also sketch a diagram.

$tan3.4° = y/300ft \qquad y \cong 17.8\, feet$

7. You are on the NE corner of 24th St and 9th Avenue in Manhattan looking at the building known as London Terrace. You paced it off and estimate that you are 120 feet from the building. Using your phone you measure the angle to the top at 63.5º. How tall is the building?



ca. 240 ft.

8. What will the angle be if you move back another 120 ft?

Both legs of the right triangle will be equal. Thus 45º.

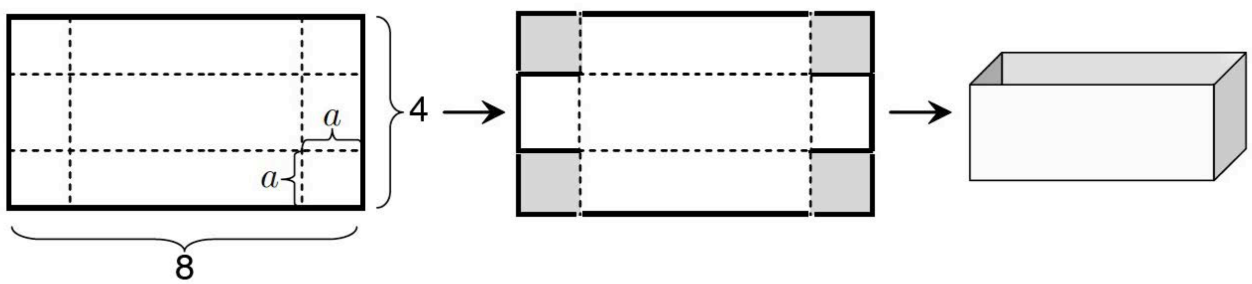
9. You are standing in a western suburb of Yamoussoukro, the capital of Ivory Coast (Côte d'Ivoire) and you notice that from where you are standing you see an airplane eclipsing the north star. You know that you are exacty 6 miles from the air port and the plane is traveling west to east. Estimate the altitude (in feet) of the airplane.

Lat. 6.8º N. 0.715 miles high, or 3778 ft.

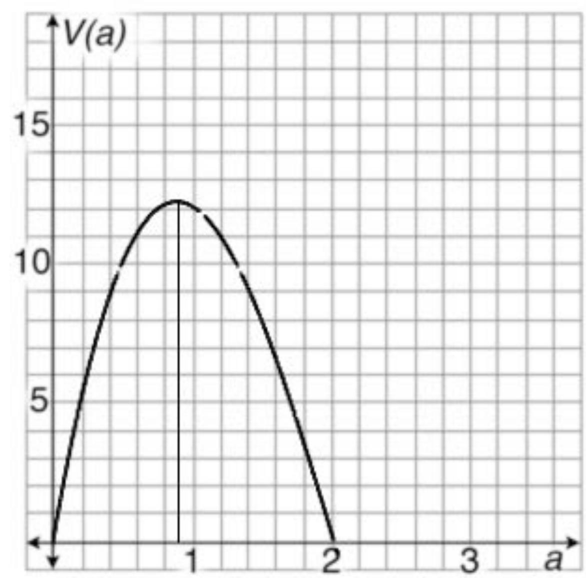
10.

5) How does a 100 mph (mile per hour) velocity break up into x and y components if the angle off of horizontal is:

Angle	x-velocity	y-velocity
0º	100	0
30º	86.6	50
45º	70.7	70.7
60º	50	86.6
90º	0	100



11) You have an $8' \times 4'$ piece of cardboard. You are going to cut-score-and-fold an $a \times a$ square out of each corner in order to fold it into a box (with no top). The volume will be *base-times-length-times-height*.



a) Write the volume as a function of *a*. *V(a)* = ?

V(a) = a(8 - 2a)(4 - 2a) = 2a(4 - a)2(2 - a) = 4a(4 - a)(2 - a) =
V(a) = 4a(8 - 6a + a^2)
V(a) = = 4a(a - 2)(a - 4)
The zeros are at 0, 2, and 4.

b) Sketch a graph of the formula from part a, between *a* = *0* and *a* = *2*.

c) Use the cubic vertex formula to find max/min: $\frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$ on the form $ax^3 + bx^2 + cx$.

Max at ca. 0.845. (for a volume of ca. 12.3 cubic feet)
The source of the cubic max/min equation is from the 1st derivative of the form: $3ax^2 + 2bx + c$... then turned into a quadratic formula... finding the zero-slope regions of the cubic. These are where the maxima and minima are... where the slope is 0.

EXTRA-CREDIT: Start with a cubic equation $f(x) = ax^3 + bx^2 + cx$.

The first derivative of that cubic is $f(x)' = 3ax^2 + 2bx + c$.

The first derivative, f(x)', is the slope of the cubic equation at any given *x*. The maximum or minimum of a curve has a slope of 0. Thus, the first derivative of a cubic equation gives you an easy way to find the max or min of a cubic. Just find the zeros.

The first derivative of a cubic equation is a quadratic equation, and we know how to find the zeros of quadratic equations. Use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Your mission is to plug the first derivative [$y' = 3ax^2 + 2bx + c$] into the quadratic equation and get it into this this form: $\frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$. This is a machine to find the maximums and minimums of a cubic.