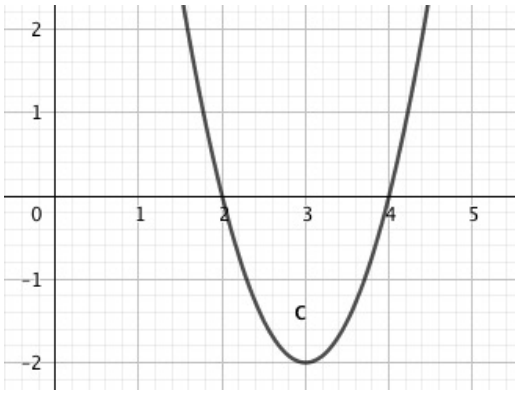
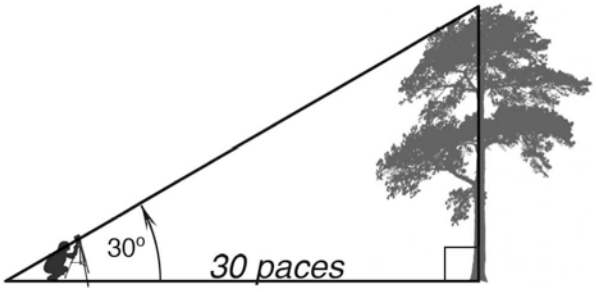
Finding the vertex of a quadratic equation. This is a relatively intuitive way. However, this derivation is very messy and I would recommend looking at the example first.	Example. This example is much easier to follow than the derivation on the left. I recommend you use this example as a model for doing the problems.
Change the standard form... $y = ax^2 + bx + c$ to the vertex form ... $y = a(x - h)^2 + k.$ *By completing the square.* *Clean up the ax^2 in order to do this.* $y/a = x^2 + (b/a)x + c/a$ $h = (b/a)/2 = b/2a$ *and* $h^2 = [(b/a)/2]^2 = (b/2a)^2$ *Now add and subtract h^2 from the right-hand side.* $y/a = \mathbf{x^2 + (b/a)x + (b/2a)^2} - (b/2a)^2 + c/a$ *Factor the part in **bold and underlined**...* $y/a = (x + b/2a)^2 - (b/2a)^2 + c/a$ *And clean up the y and rearrange.* $y = a(x + b/2a)^2 + [c - (b^2/4a)]$ *Now you can see how to graph this equation.* ***The vertex is at $x = -b/2a$ and $y = c - (b^2/4a)$***	$y = 2x^2 - 12x + 16$ *Get it into this form in order to complete the square by dividing both sides by 2.* $y/2 = x^2 - 6x + 8$ *Now think of this as in the form: $x^2 + bx + c$* *That makes $h = b/2 = -3$ and $h^2 = (b/2)^2 = 9$* *Now add and subtract 9 from the right-hand side.* $y/2 = x^2 - 6x + 9 - 9 + 8$ *Factor it...* $y/2 = (x - 3)^2 - 1$ *And clean up the y.* $y = 2(x - 3)^2 - 2$ *Now you can see how to graph it.* **The vertex is (3, -2)**

Recall, this is just the UR-quadratic, x^2,
shifted to the right 3 and down 2. It's happy because the squared stuff is positive. Therefore, this vertex is a minimum. A happy quadratic has a minimum point. A sad quadratic has a maximum point.

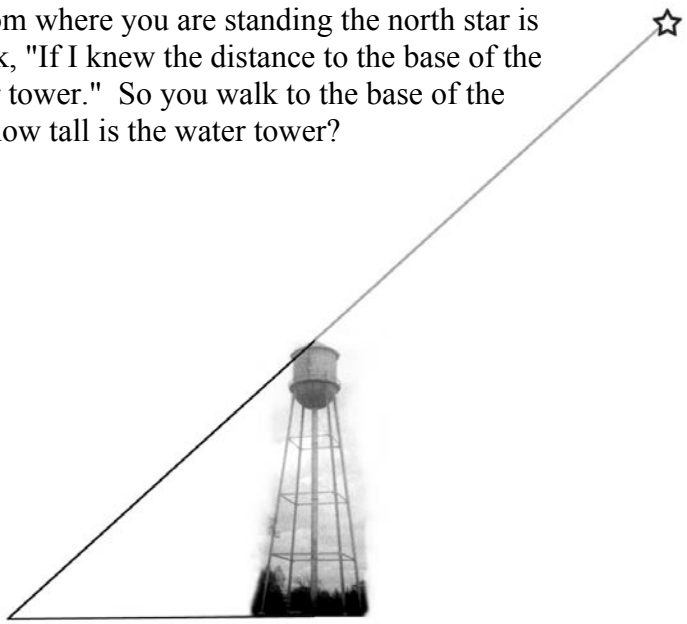


Finding the vertex of a quadratic equation. The less intuitive, but more direct, way.	Example
Change the standard form... $f(x) = ax^2 + bx + c$ to the vertex form $f(x) = a(x - h)^2 + k,$ ...which is constructed by completing square... where $h = \frac{-b}{2a}$ and $k = f(h)$ The vertex will be (h, k)	$f(x) = 2x^2 - 12x + 16$ $a = 2 \quad b = -12 \quad c = 16$ $h = \frac{12}{2 \cdot 2} = 3$ $k = f(3) = 18 - 36 + 16 = -2$ *The vertex form is:* $f(x) = 2(x - 3)^2 - 2$ The vertex is (h, k) = (3, -2)

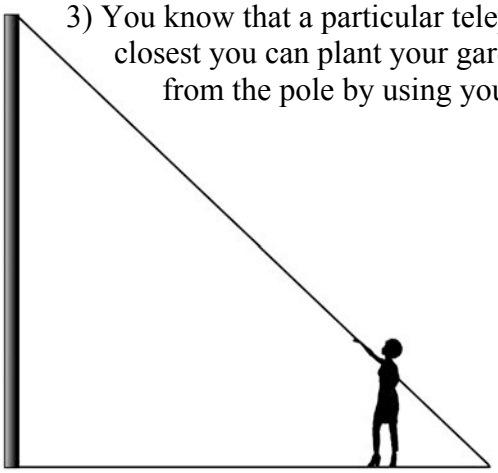
1) How tall is the tree (in feet)? [1 pace is 30".]



2) You are standing in Tivoli and you notice that from where you are standing the north star is exactly at the top of the water tower. You then think, "If I knew the distance to the base of the water tower, I could estimate the height of the water tower." So you walk to the base of the water tower and count 44.5 paces. Approximately how tall is the water tower?
[1 pace is 30".]



3) You know that a particular telephone pole is 40' tall. The electric company told you that the closest you can plant your garden is 40' from the pole. How can you determine your distance from the pole by using your arm to measure the angle? Guestimate.



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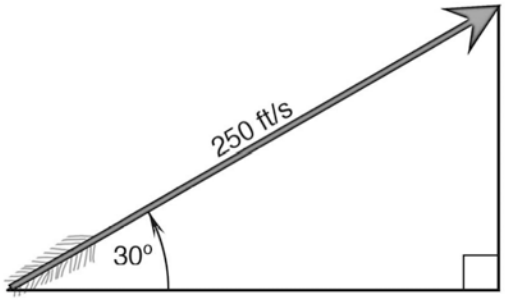
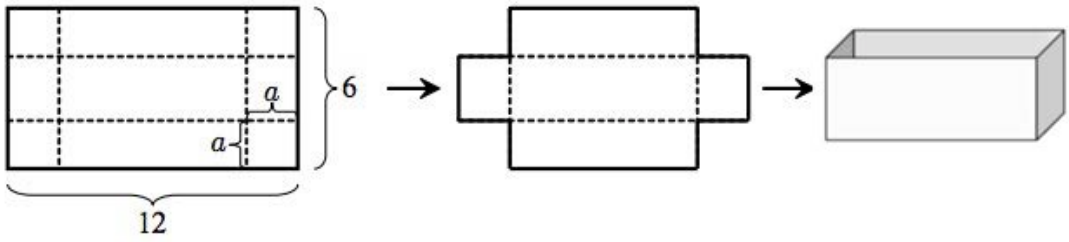
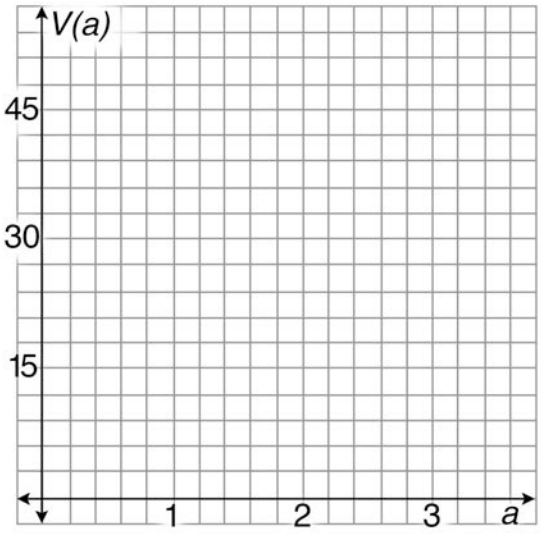
4) Odysseus shoots an arrow from his incredibly mighty bow. What are the vertical and horizontal components of a *250 ft/s* velocity directed 30º upward as shown in the diagram.





5) You have an $12' \times 6'$ piece of cardboard. You are going to cut-score-and-fold an $a \times a$ square out of each corner in order to fold it into a box (with no top).
The volume will be *base-times-length-times-height*.



a) Write the volume as a function of *a*. *V(a)* = ?

b) Sketch a graph of the formula from part a, between $a = 0$ and $a = 2$.

c) Use the **cubic vertex formula** to find max/min: $\frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$
on the form $ax^3 + bx^2 + cx$.