A SYMBOL OF EQUALITY

by D. Newsome

DATES	SOURCES	1	2	3	4	5	6	7	8	9	0
976	Spain: Codex *Vigilanus*. Escurial, Ms. lat. d.l.2, f° 9v	1	2	3	4	5	6	7	8	9	
992	Spain: Codex *Aemilianensis*. Escurial, Ms. lat. dl.1, f° 9v	1	2	3	4	5	6	7	8	9	
Before 1030	France (Limoges). Paris, BN Ms. lat. 7231, f° 85v	1	2	3	4	5	6	7	8	9	
1077	Vatican Library, Ms. lat. 3101, f° 53v	1	2	3	4	5	6	7	8	9	
XIth C	Bernelinus, *Abacus*. Montpellier, Library of the Ecole de Médecine, Ms. 491, f° 79	1	2	3	4	5	6	7	8	9	
1049?	Erlangen, Ms. lat. 288, f° 4	1	2	3	4	5	6	7	8	9	
XIth C	Montpellier, Library of the Ecole de Médecine, Ms. 491, f° 79	1	2	3	4	5	6	7	8	9	
XIth C	France: Gerbertus, *Raciones numerorum Abaci*. Paris, BN Ms. lat. 8663, f° 49v	1	2	3	4	5	6	7	8	9	
XIth/ XIIth C	Lorraine: Boecius, *Geometry*. Paris, BN, Ms. lat. 7377, f° 25v	1	2	3	4	5	6	7	8	9	
XIth C	Boecius, *Geometry*. London, BM, Ms. Harl. 3595, f° 62	1	2	3	4	5	6	7	8	9	
XIth C	Germany (Regensburg). Munich, Bayerische Staatsbibliothek, Clm 12567, f° 8	1	2	3	4	5	6	7	8	9	
XIth C	Boecius, *Geometry*. Chartres, Ms. 498. f° 160	1	2	3	4	5	6	7	8	9	
Early XIIth C	Bernelinus, *Abacus*. London, BM Add. Ms. 17808, f°, 57	1	2	3	4	5	6	7	8	9	
Late XIth C	Bernelinus, *Abacus*. Paris, BN Ms. lat. 7193, f° 2	1	2	3	4	5	6	7	8	9	
Late XIth C	France (Chartres?): Anon., Arithmetical tables. Paris, BN Ms. lat. 9377, f° 113	1	2	3	4	5	6	7	8	9	
Late XIth C	Bernelinus, *Abacus*. Paris, BN Ms. lat. 7193, f° 2	1	2	3	4	5	6	7	8	9	
XIIth C	Rome, Alessandrina Library, Ms. 171, f° 1	1	2	3	4	5	6	7	8	9	
XIIth C	Paris, Saint Victor. Gerlandus, *De Abaco*. Paris, BN, Ms. lat. 15119, f° 1	1	2	3	4	5	6	7	8	9	
XIIth C	Boecius, *Geometry*. Paris, BN, Ms. lat. 7185, f° 70		2	3	4	5	6	7	8	9	
XIIth C	France, Chartres(?):Bernelinus, *Abacus*. Oxford, Bodley, Ms Auct. F. l. 9, f° 67v	1	2	3	4	5	6	7	8	9	
XIIth C	Gerlandus, *De Abaco*. London, BM Add. Ms. 22414, f° 5	1	2	3	4	5	6	7	8	9	
XIIth C	Gerlandus, *De Abaco*. Paris, BN Ms. lat. 95, f° 150	1	2	3	4	5	6	7	8	9	
Early XIIIth C	France (Chartres): Anon. Paris, BN Ms. lat. Fonds Saint-Victor, 533, f° 22v	1	2	3	4	5	6	7	8	9	

FIG. 26.4. *Mediaeval* apices. *Sources: BSMF X (1877), p. 596; Burnam (1920), II, plates XXIII & XXIV; Folkerts (1970) Friedlein (1867) p. 397; Hill (1915) Smith and Karpinski (1911) p. 88*

The symbols used in mathematics are relatively modern inventions, even the numbers themselves. [See figure at left.[1]] For most of human history, mathematics was written in prose. "Three added to ten results in thirteen." But algebra signals a shift. It distills quantifiable ideas down to a symbolic language written especially for the purpose of discussing mathematics.

Algebra originated in 9th-century Baghdad in a book called, الجبر والمقابلة. [*al-Kitāb al-mukhtaṣar fī ḥisāb al-jabr wal-muqābala*]. It was written by the Persian, Al-Khwarizmi. [The *kh* is pronounced like a Spanish *jota*. His name should probably be pronounced *al-guarismi*, a bit like algorithm. In fact, a Spanish word for "numeral" is *guarismo… khwarizmi*.]

This book was imported into Europe and translated into Latin in the 12th century. The word, "algebra," refers to "*al-jabr*" seen in the Latinized title of the book [above]. "*Al-jabr*" literally refers to the process of making a negative term on one side of an equation into a positive term on the other. The other operational term in the title, *al-mukhtaṣar*, refers to cancelling like terms on both sides of an equation. Both processes are familiar to all students of algebra.[2]

Algebra is based, first and foremost, on equality. You can't do algebra without equality. But writing down the concept of equality has only recently become standardized.

A Short History of Symbolic Equality

Ancient Egyptian equality was written in prose as, ♎, and translates from hieroglyphics to, "it gives."[3] The Greek, Diophantus (fl. ca. 250 A.D.), used this symbol, ι°, to mean equality.[4] The word, *pha*, was used for equality in a 10th century manuscript from India.[5] Al-Qalasadi (15th century,

[1] Ifrah, Georges. *The Universal History of Numbers from Prehistory to the Invention of the Computer*. New York: Wiley, 2000.

[2] Karpinski, Louis C. "Algebra." *Modern Language Notes* XXVIII (March 1913): 93.

[3] Peet, T. Eric, ed. *The Rhind Mathematical Papyrus*, British Museum, Nos. 10057 and 10058, Introduction, Transcription, and Commentary (London, 1923). Sometimes called the Ahmes Papyrus. Dated to ca. 1550 B.C. Described in Cajori, Florian. *A History of Mathematical Notations: Two Volumes Bound as One*. New York: Dover, 1993; §23

[4] Cajori, §103.

[5] From the Bakhshali Manuscript. This manuscript is interesting in many ways. It uses a place-system for decimal notation. Instead of a 0 [zero], like we use, they used a dot. E.g. 307 was written 3·7. And if that is not confusing enough, they used a "+" to mean minus. Cajori, §109.

Iberian) used this symbol, ﾉ , for equality.[6] Regiomontanus (d. 1476. Italian) used a horizontal line much like we do when doing vertically aligned arithmetic. E.g. 21 and 5 / 26 or 21 / + 5 / 26

He also wrote out "and" for plus [*et* in Latin] and "less" for subtraction [*minus* in Latin].[7] Pacioli [late 15th century, Italy] took this same line [see above], using it more or less like Regiomontanus, but he also used it horizontally. E.g. 2 plus 6 ——— 8. Cardano [mid 16th century, Italian] either wrote out the word equals [*aequalia* in Latin] or left a big space with a result following.[8] E.g. 1 plus 18 19. Finally in the mid-16th century we get to the Welsh mathematician, Robert Recorde, who first used the familiar = to mean equality. The symbol debuted in 1557 but it didn't really catch on at first. It appeared again in print in 1618 in an anonymous appendix to a book by Napier, but still found few followers. Only in 1631 did it really take off. In that year it appeared in three significant mathematical books by Harriot, Oughtred, and Norwood (all Englishmen).[9]

One of the reasons the = didn't catch on was that the symbol was simultaneously being used for other concepts. In 1591 François Viète used it to mean minus. In 1638 Descartes used it to mean ±. Another author in 1670, Caramuel, used it as we now use a decimal point. E.g. 102=857 meant 102.857. Adding insult to injury, the = symbol was used frequently to designate parallel lines. There was little agreement for what = meant. Similarly, other symbols were competing to become equality, such as [. E.g. 2 + 3 [5. Another popular one was ‖. Another was simply |. In the late 16th century the famous popularizers of Copernicus, Leonard and Thomas Digges, used ⋕ for equality. [My personal favorite.] Other contenders found in 17th-century texts include ⊔ , ⊓ , ~, ∞, and {, One curious example rotated the number 11 to, ⥣.

The biggest threat to = was the ♾, which Descartes used prominently in his wildly influential book, *Geometrie,* from 1637. Descartes' symbol for equality appears to be the symbol for Taurus, ♉ , rotated 90º counter clockwise. Any printer who worked with mathematical texts would have had type for this symbol. Astrology was, after all, considered part of mathematics back then. For some of you, it may be surprising to learn that the evolution of the equals sign is somehow tied up in astrology.

Throughout the latter 17th century the Cartesian symbol [the rotated Taurus] was widely used, especially in Holland and France, whereas in England, Recorde's = dominated. But then some heavy hitters started using Recorde's symbol. Isaac Newton used it consistently as did Newton's arch-enemy Leibniz, who used it intermittently. Even Descartes occasionally used = in his private correspondence. Generally speaking the ♾ dominated continental Europe into the early 18th century, but eventually Recorde's = won. And it won big. Few mathematical symbols have become so universal. Symbols for multiplication, division, and roots still vary considerably from text to text. But equality is =.

Similar essays could trace the development of most of the other modern algebraic symbols.

Exercises: A lot of doing algebra is recognizing patterns and context. You have to figure things out. You have to infer meanings. Below are a few examples of mathematical thought.

A) 4 and 3 @ 7.	What does @ mean?	@ probably means equals.
B) 6 ♉ 7 = 42.	What does ♉ mean?	
C) 4 // 3 equals 1.	What does // mean?	
D) 1 + & = 3.	What is &?	
E) 1 ▲ 2 ▲ 3 ▲ 4 ♾ 10.	What is ▲? What is ♾?	
F) 1 ☞ 2 ☞ 3 ⋕ 6.	What is ☞? What is ⋕?	

[6] From Al-Qalasadi's *Raising of the Veil of the Science of Gubar.* Cajori, §124.
[7] Cajori §126.
[8] Cajori §140.
[9] Cajori §261. The remainder of this essay is based on Cajori §260-270.