Name: _____

Exercise 1: A Few Different Ways to Model Population Changes

Figure out the algorithm that created the following population data
or fill in the populations from the given algorithm.

	E.g.	a.	b.	c.	d.	e.	f.
Year	Population	Population	Population	Population	Population	Population	Population
0	10	250				4800	100
1	15	295		200		2400	125
2	20	340				1200	156.25
3	25	385	700		16384	600	195.31
4	30	430				300	244.14
5	35	475				150	305.18
6	40	520	400	6400		75	381.47

E.g. Write P(t), population as a function of time (in years). See population data for "E.g." above.

Answer:

$$P(t) = 5t + 10$$

a) Write P(t), population as a function of time (in years).

Answer:

b) Fill in the Populations on the table above using the following function:

$$P(t) = 1000 - 1000t/10 = 1000(1 - t/10)$$

c) Fill in the Populations on the table above using the following function:

$$P(t) = 100(2^t)$$

d) Fill in the Populations on the table above using the following function:

$$P(t) = 1{,}048{,}576(0.5)^{2t}$$

e) Write P(t), population as a function of time (in years).

Answer:

f) Write P(t), population as a function of time (in years). The population data was rounded to two places.

Answer:

Exercise 2: The S-I Model (Susceptible-Infected Model)

What we need for this model are four inputs:

i, I_0, S_0, and T

[*i*nfection rate, initial *I*nfected population, initial *S*usceptible population, and the *T*otal population.]

The general equation is: $I_1 = i(I_0)\left(\frac{S_0}{T}\right)$.

The additional infected population produced on Day-1 [I_1] is the infection rate times the initial Infected population times the initial Susceptible population divided by the Total population.

E.g.1

i = ½ per day *I_0 = 16* *S_0 = 100* *T = 116*

This set of initial conditions, (Day-0 conditions) means that everybody, except for the 16 Infecteds, are Susceptible and that the Infected population have a virus with an infection rate of ½ of person per day. All tolled, the Total population is 116 (the Infected plus the Susceptible).

$$I_1 = i(I_0)\left(\frac{S_0}{T}\right) = \frac{1}{2}(16)\left(\frac{100}{116}\right) = \frac{800}{116} \approx 6.9\ people.$$

These conditions produce an additional 6.9 infected people.

The Total accumulated Infected population on Day-1 [I_{T1}] will be the sum of the initial Infected population plus the newly infected from Day-1.

$$I_{T1} = I_0 + I_1 = I_0 + i(I_0)\left(\frac{S_0}{T}\right) \approx 16 + 6.9 = 22.9\ people.$$

E.g.2

i = ½ per day *I_0 = 16* *S_0 = 100* *T = 216*

This set of initial conditions means that Total population is made up of 16 Infected, 100 Susceptible, and 100 unSusceptible (immune for whatever reason).

$$I_1 = i(I_0)\left(\frac{S_0}{T}\right) = \frac{1}{2}(16)\left(\frac{100}{216}\right) = \frac{800}{216} \approx 3.7\ people.$$

These conditions produce an additional 3.7 infected people on Day-1 for an accumulated total of infected people, I_{T1}:

$$I_{T1} = I_0 + I_1 = I_0 + i(I_0)\left(\frac{S_0}{T}\right) \approx 16 + 3.7 = 19.7\ people.$$

Your turn... Fill in the empty boxes.
Round all answers to nearest 1000th. I did a few so that you can verify if you are doing this correctly.

One day of infection						$I_1=i(I_0)(S_0)/T$	$I_{T1}=I_0+I_1$
i	***I_0***	***S_0***	***T***	***unSusceptible***	***S_0/T***	***I_1***	***I_{T1}***
4	1	1000	1001	0	0.999		4.996
400%	16	1000	2016	1000	0.496		
2	1	1000	1001				
200%	16	1000	2016				31.873
1	1	1000	1001			0.999	
100%	16	1000	2016				
0.5	1	1000	1001				
50%	16	1000	2016				
0.25	1	1000	1001				
25%	16	1000	2016				
5%	1	1000	1001			0.050	
5%	16	1000	2016				16.397
2%	1	1000	1001	0	0.999		
2%	16	1000	2016	1000	0.496		
1%	1	1000	1001				
1%	16	1000	2016				16.079

Because this table is a bit of a pain to fill out, I highly suggest you do this on a spreadsheet like Excel, GoogleSheets, or Apple Numbers.

Here are the formulas for the first 2 rows: (Formulas in red.)
All of the rest of the stuff in there is either data or labels.

COLUMN→	B	C	D	E	F	G	H	I
ROW↓							$I_1=i(I_0)(S_0)/T$	$I_{T1}=I_0+I_1$
4	***i***	***I_0***	***S_0***	***T***	***unSusceptible***	***S_0/T***	***I_1***	***I_{T1}***
5	4	1	1000	1001	=E5-D5-C5	=D5/E5	=B5*C5*D5/E5	=H5+C5
6	4	16	1000	2016	=E6-D6-C6	=D6/E6	=B6*C6*D6/E6	=H6+C6

Etc.