Name________________________

BLC190- ATF-Fall-2020

Homework 8: GeoTrig

Circumference of a Circle: $\pi d = \pi 2r = 2\pi r$
(all the same since $d = 2r$)

If r = 1, then the circumference is just 2π. All the way around a circle of radius 1 is 2π.

That's what is meant when math teachers say that 2π radians is 360º.
[Radians is usually abbreviated "rads."]

Radians are a way of measuring angles using the circumference of a circle, but instead of measuring the portion of the circumference in inches or centimeters, you measure it by its own radius, called radians in this context.

If 2π rads = 360º,

then $1\ rad = \frac{360°}{2\pi} = \frac{180°}{\pi}$

and

$$1° = \frac{2\pi\ rads}{360} = \frac{\pi\ rads}{180}$$

The conversion equations:

$$\mathbf{1\ rad = \frac{180°}{\pi}} \quad \textbf{and} \quad \mathbf{1° = \frac{\pi\ rads}{180}}$$

See Homework Exercise 1.

Doing Geometry to Establish Trigonometry

In Class Exercise: Geometrical Construction Exercises: Play with the compass and straight edge and practice making circles, equilateral triangles, squares, right angles, finding midpoints, bisecting angles, and transferring measurements.

I. Make a pretty picture.

a. First, fold a sheet of paper in half to create a base line.
b. Now I want you to draw a circle with the center on the center of that line with a radius of about 2" (5cm).
c. Then go nuts with your compass by drawing circles centered on points of intersection... where-ever two lines cross, use that cross as a center for a new circle. All circles must have the same radius, so don't fiddle with your compass after the first circle. Make at least 6 or 7 circles.... preferably more. Make some sort of pretty design. Go nuts.
d. Now pull out a straight edge and see if you can find equilateral triangles. Use the straight edge to connect the dots for these triangles.
e. Can you find a regular hexagon? [6-equal-sided polygon]
f. Feel free to color/shade in parts of your picture. Highlight the triangles and hexagons and any other interesting shape you like. Make it pretty.

II. Figure out the simplest method for the construction of an equilateral triangle. [Hint: Start with a straight line (you can use a fold) and then make circles with centers on that line.]

II.1. Optional. Figure out the simplest way to make a regular hexagon.

III. If you figured out part II, you should see that this construction also leads to the following:

a. Finding the midpoint of a line segment.
b. Bisecting an angle.
c. Creating a line perpendicular to another line.

Convince yourself of these constructions. See Homework Exercise 2.

Exercise 1: Converting from degrees to rads. and vice versa. To be turned in next week.

E.g. 3 rads = ?º $$1\,rad = \frac{180°}{\pi} \quad \rightarrow \quad 3\,rads = \frac{3 \cdot 180°}{\pi} = \frac{540°}{\pi} \cong 171.89°$$
E.g. 15º = ? rads. $$1° = \frac{\pi\,rads}{180} \quad \rightarrow \quad 15° = \frac{15 \cdot \pi\,rads}{180} = \frac{\pi\,rads}{12} \cong 0.263\,rads$$
1. 30º = ? rads.
2. 3.1415... rads = ?º
3. 45º = ? rads.
4. 3π rads. = ?º
5. 90º = ? rads.
6. $\frac{\pi}{2}$ rads. = ? °

Exercise 2: Geometric constructions.

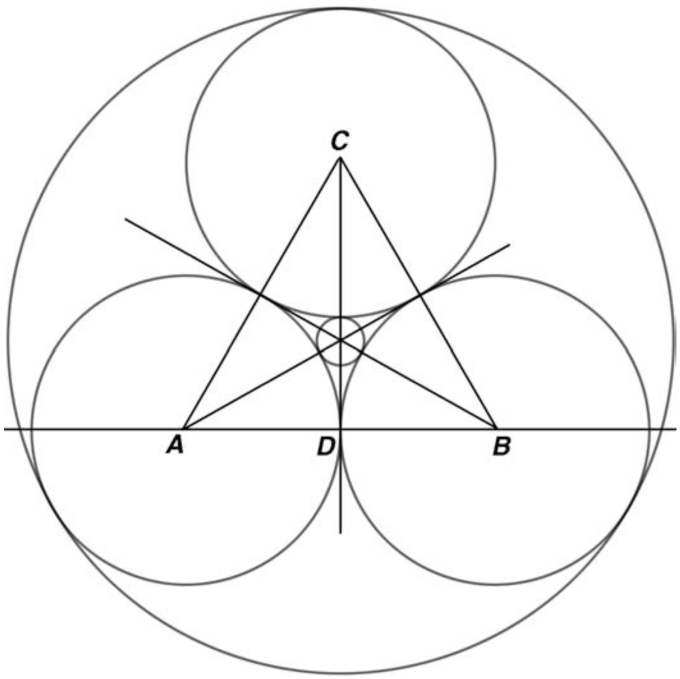
1) Describe in prose with diagrams how to construct an equilateral triangle. Make this good enough to teach a 5th grader. I want a beautiful, finished page. It could look like a standard textbook, or a comicbook, or whatever you want, but teach a kid how to make an equilateral triangle.

2) Geometrically construct an equilateral triangle with a circle inscribed inside it and another circle circumscribed around it. Make a nice diagram to turn in.

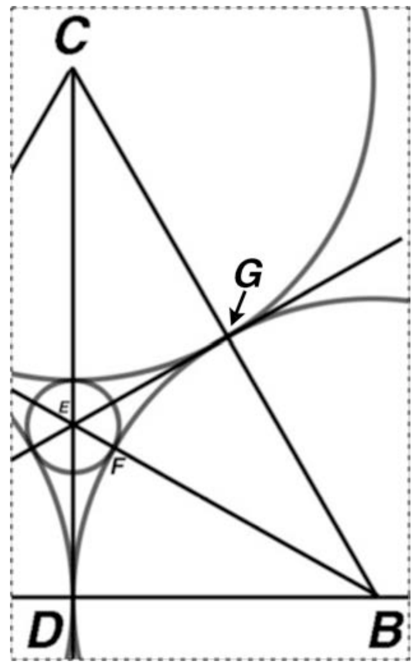
3) Geometrically construct a square with a circle similarly circumscribed around and it and another one inscribed in it. Make a nice diagram to turn in.

Exercise 3: The Pythagorean Theorem and Trigonometry. To be turned in next week.

Set-up for Exercise 3. We'll go over the details in class.



The main diagram.



Detail of the main diagram.

Given:

- -CB is 2.
- -ABC is an equilateral triangle. (meaning that all interior angles are 60º and all sides are 2.)
- -The angles at A, B, and C have been (equally) bisected with the lines shown. E.g. CD bisects the angle ACB.
- -These bisection lines are perpendicular to the sides opposite the bisected angles. E.g. Angle CDB is a right angle.
- -These bisection lines also intersect the sides of the equilateral triangle at the midpoints of each side. E.g. Point D is the midpoint of the line segment AB.
- -The three medium-sized circles are centered on points A, B, and C.
- -The three interior angles of a triangle add up to 180º.
- -The Pythagorean Theorem.
- -Here's the kicker... this is the thing you need to know in order to solve this problem: If two angles of a triangle have measures equal to the measures of two angles of another triangle, then the triangles are <u>similar</u>. If one triangle is similar to another, it means that one is a scaled version of the other one.

Let's say you have two similar triangles. If you know the lengths of the sides of one triangle, you can deduce the lengths of the sides of the other triangle so long as you know the length of one of its sides.
For example, if two triangles, A and B, are similar, and Triangle A has a hypotenuse of 6 and the Triangle B has a hypotenuse of 3, you can deduce that Triangle B's legs are half the length of Triangle A's legs.

Hint: What is the relationship between triangles CDB and BDE?

Assuming all the things given above, figure out all of the measurements below.

Table A.

Angle	In Degrees	In Radians
$\angle DCB$		
$\angle CBD$		
$\angle CDB$	90	$\frac{\pi}{2}$
$\angle EBD$		
$\angle BED$		
$\angle CGE$		

Angle	In Degrees	In Radians
$\angle GBE$		
$\angle GEB$		
$\angle EGB$		
$\angle GCE$	30	$\frac{\pi}{6}$
$\angle GEC$		
$\angle CGE$		

Table B.	⊿BDC	⊿EDB	⊿EGC	⊿EGB
short leg		$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$		
long leg				
hypotenuse	BC = 2			

Given the information in the table above, what is the scaling factor to get from ⊿BDC than ⊿EDB? $\sqrt{3}$

Table C.

What is the ratio of $\overline{BC}:\overline{BE}$?	
What is the ratio of $\overline{CD}:\overline{BD}$?	
What is the ratio of $\overline{BD}:\overline{DE}$?	

What is the ratio of $\overline{CE}:\overline{DE}$?	2:1
What is the length of $\overline{BF}$?	
What is the length of $\overline{EF}$?	

Table D.

$\sin(\angle DCB) =$	
$\cos(\angle DCB) =$	$\sqrt{3}/2$
$\tan(\angle DCB) =$	

$\sin(\angle CBD) =$	
$\cos(\angle CBD) =$	
$\tan(\angle CBD) =$	

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