

Homework 9- Linear Equations and Graphs, pt. II

The slope equation does a lot of work. It finds the slope from 2 points.

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

The input data is from two points, (x_1, y_1) and (x_2, y_2) . But you can also input a single point and a slope and find the $y = mx + b$ form for the equation of this line.

<p>Assume there is a line.</p> <p>Let's say you know a point, (x, y), and a slope, m.</p> <p>Find the $y = mx + b$ form for the equation of this line.</p>	<p>$(2, 3)$ is the point and $m = 3$ for the slope.</p>
<p>Simply plug what you know into the slope formula.</p> <p>I chose to assign my known point $(2, 3)$ to be (x_1, y_1), but it actually doesn't matter. I'll show that below.</p>	$m = \frac{y_2 - y_1}{x_2 - x_1} =$ $3 = \frac{3 - y_1}{2 - x_1}$
<p>Solve this equation for the unknown y, in this case, y_1. Perhaps the easiest way to do this is to cross multiply.</p>	$\frac{3}{1} = \frac{3 - y_1}{2 - x_1} \rightarrow 3 - y_1 = 3(2 - x_1)$
<p>Now just multiply things out and get the y by itself.</p> <p>... multiply both sides by -1 to get rid of that negative sign.</p> <p>And that's it. You have the $y = mx + b$ form and can read off the slope and $y_{\text{intercept}}$. $m = 3$ and $y_{\text{int.}} = -3$ [...or $(0, -3)$]</p>	$-y_1 = 6 - 3x_1 - 3$ $-y_1 = -3x_1 + 3$ $y_1 = 3x_1 - 3$
<div data-bbox="45 1155 387 1584" data-label="Figure"> </div> <p>Just to be complete, this time I'll assign (x_2, y_2) to the known point. Cross multiply, simplify, etc. Unsurprisingly, you get the same linear equation. And you'll notice that the point we were given, $(2, 3)$ is on this line. [It had better be.]</p>	$m = \frac{y_2 - y_1}{x_2 - x_1} =$ $3 = \frac{y_2 - 3}{x_2 - 2} \rightarrow y_2 - 3 = 3(x_2 - 2)$ $y_2 - 3 = 3x_2 - 6$ $y_2 = 3x_2 - 3$

Exercise 1: Figure out the equation of the line in $y = mx + b$ form using the information provided and then graph the line on a graph that you have to draw. [Label axes and add some numbers to show scale.]

E.g.
 (2, 3) and (0, -3)
 An alternate method:
 Find slope, $m = 3$
 Then plug your slope and one of your points into $y = mx + b$... like so
 $y = mx + b$
 $3 = 3(2) + b = 6 + b$
 solve for b
 $b = -3$
 Now put the m & b into $y = mx + b$ → $y = 3x - 3$

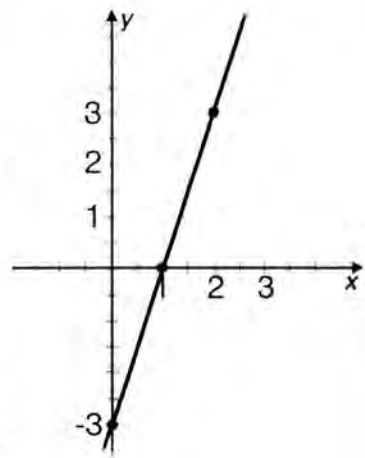
You have 2 points. First find the slope.
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-3)}{2 - 0} = \frac{6}{2} = 3$
 Then plug the slope and one of your known points back into the slope equation and solve for the y term. I'm using the second point.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow 3 = \frac{y_2 - (-3)}{x_2 - 0} \rightarrow$$

$$\rightarrow \frac{3}{1} = \frac{y_2 + 3}{x_2} \rightarrow y_2 + 3 = 3x_2 \rightarrow$$

$$\rightarrow y_2 = 3x_2 - 3$$

same result → $y = 3x - 3$



You can lose the subscripts at this point. Notice that your original two points live on this line. Plug in any value for x , and this equation will output a y that lives on this line (and vice versa).

1.
 (-3, 3) = (x_1, y_1)
 and
 (3, -1) = (x_2, y_2)
 or... $y = mx + b$
 $-1 = \frac{-2}{3}(3) + b$
 $b = 1$
 $\therefore y = \frac{-2}{3}x + 1$

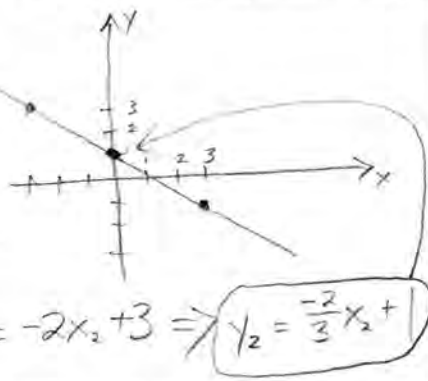
Do this just like the example above.

$$m = \frac{-1 - 3}{3 - (-3)} = \frac{-4}{6} = \frac{-2}{3}$$

$$m = \frac{-2}{3} = \frac{y_2 - 3}{x_2 - (-3)} \Rightarrow \text{cross multiply \& solve for } y$$

$$3(y_2 - 3) = -2(x_2 + 3)$$

$$3y_2 - 9 = -2x_2 - 6 \Rightarrow 3y_2 = -2x_2 + 3 \Rightarrow y_2 = \frac{-2}{3}x_2 + 1$$



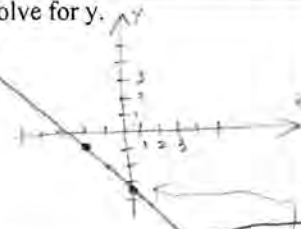
2.
 $m = -3/2$
 and
 (-1, -2)

Go straight to the slope formula and plug in the given information... solve for y .

$$m = \frac{-3}{2} = \frac{y_2 - (-2)}{x_2 - (-1)} = \frac{y_2 + 2}{x_2 + 1} = \frac{-3}{2}$$

$$2(y_2 + 2) = -3(x_2 + 1)$$

$$2y_2 + 4 = -3x_2 - 3 \Rightarrow 2y_2 = -3x_2 - 7 \Rightarrow y_2 = \frac{-3}{2}x_2 - \frac{7}{2}$$

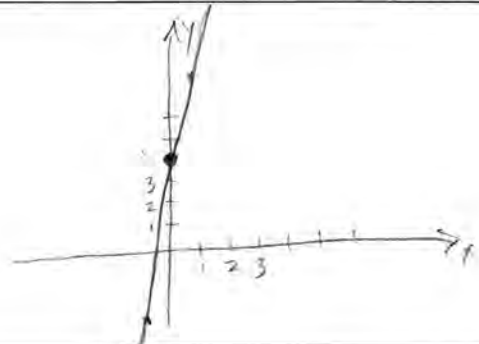


3.
 $m = 8$
 and
 (-1, -4)

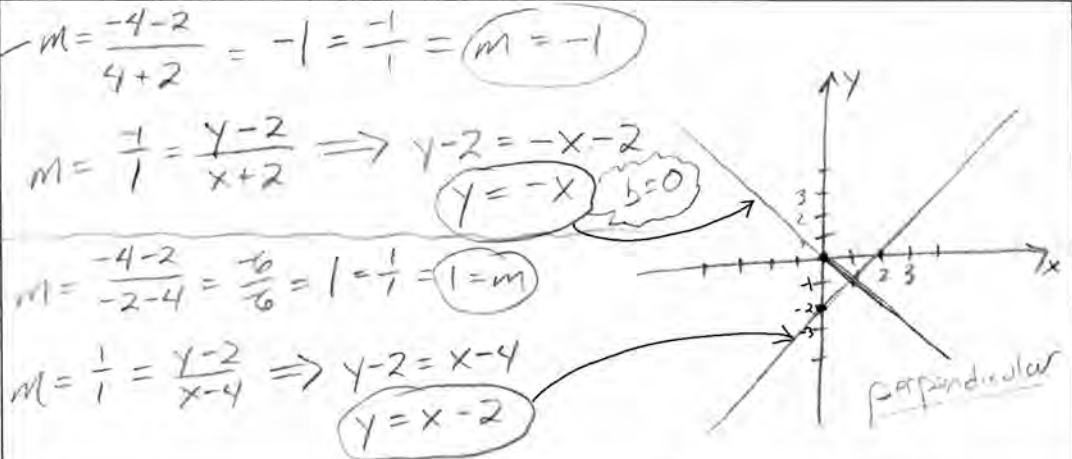
$$m = 8 = \frac{y_2 + 4}{x_2 + 1}$$

$$y_2 + 4 = 8x_2 + 8$$

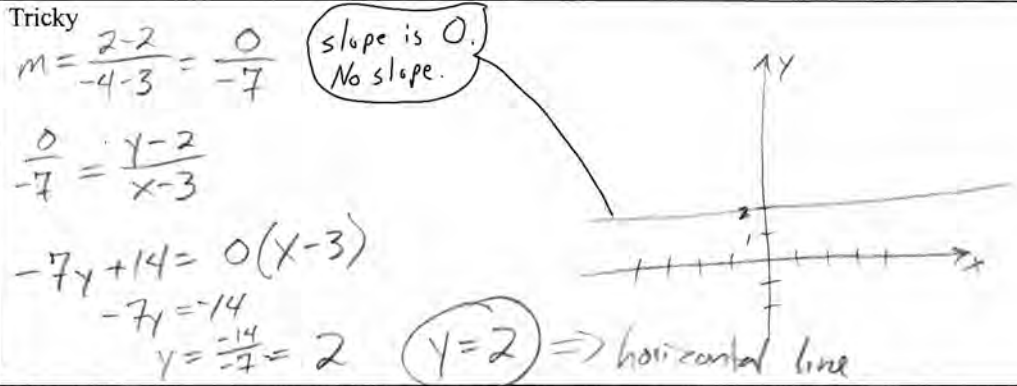
$$y_2 = 8x_2 + 4$$



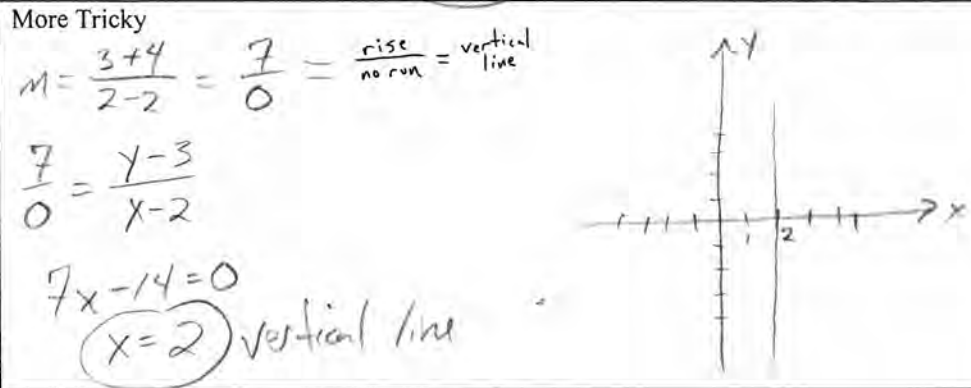
4.
 (-2, 2) and (4, -4)
 also figure out this line...
 (4, 2) and (-2, -4)
 Graph both on the same graph.



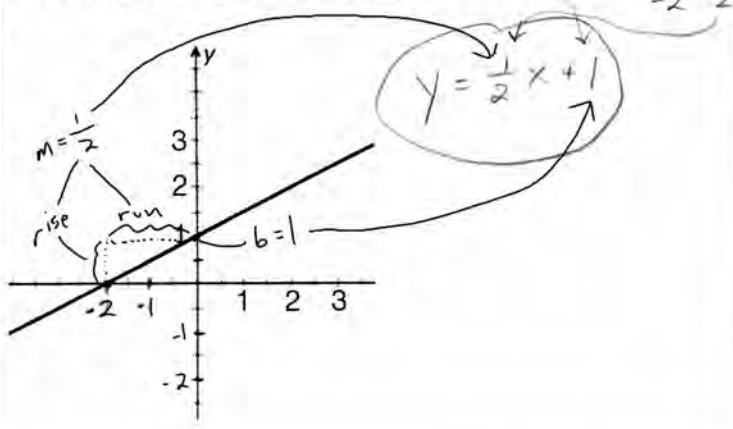
5.
 (-4, 2)
 and
 (3, 2)
 If you simply plot these 2 points you'll see what is going on.



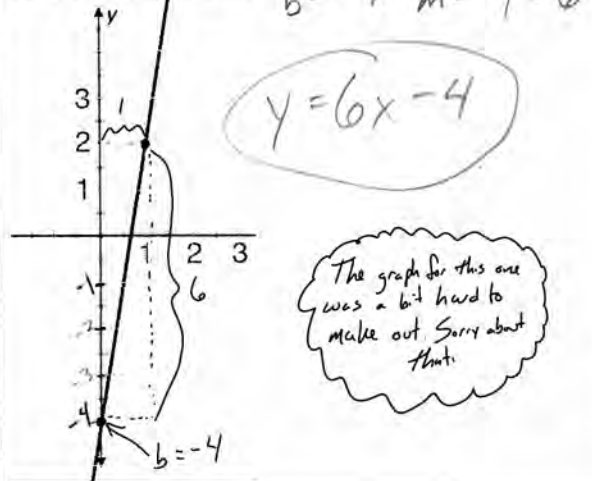
6.
 (2, -4)
 and
 (2, 3)
 plot the points d you'll see.



7. Figure out the equation of this line. Find a y-intercept and a slope, or a point and a slope.

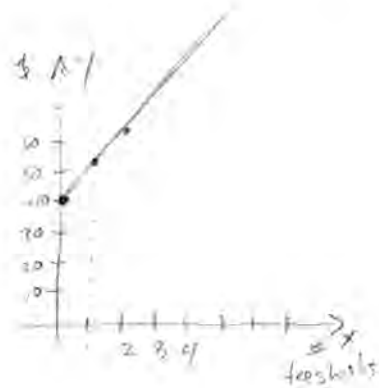


8. ... and this line... $b = -4$ $m = \frac{6}{1} = 6$



Exercise 2: A few word problems.

2.1 You are the leader of an anarchist group on campus. (Ironic, no?) You want to have some tee-shirts printed up for a march against the World Bank and the IMF in DC. The tee-shirt store says that there is a \$40 initial fee for making the silk screen and then it will charge \$12 for every tee-shirt. So, for example, if you just get 1 tee-shirt, you will have to pay \$52. Two tee-shirts will cost \$64... etc. Write a linear equation in the form $y = mx + b$ to describe this situation. The y-term will be the amount of money you will have to pay the printer and the x-term will be the number of tee-shirts you buy. Then draw a thumbnail graph of this linear equation to the right.



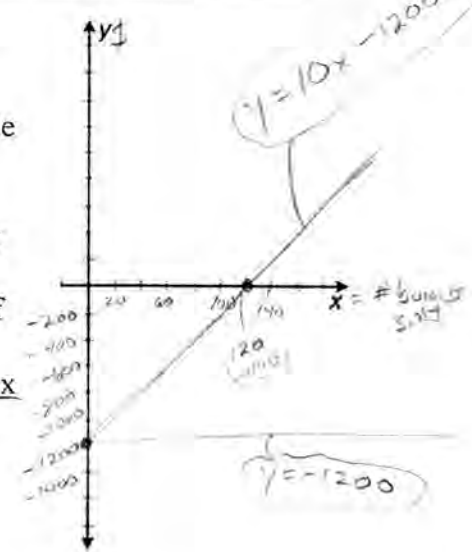
Initial fee plus x (\$12), where x is number of tee-shirts you buy.

$$y = 40 + 12x = 12x + 40$$

$m = 12$
 $b = 40$

2.2a You and a friend are starting a business selling in-vitro synthetic meat hamburgers and hotdogs at Bard baseball games. Back in 2013 a lab-grown hamburger cost \$325,000 (according to a WIRED magazine article in 2018). But the price has since plummeted. An Israeli company is currently selling synthetic meat for \$360 per pound (which includes shipping to Annandale), and each hamburger or hot dog will be a quarter pound. Your initial investment in this venture is \$1200, which will buy you a refrigerator, grilling equipment, and a cart for selling your food. If you sell each burger or dog for \$100, how many burgers/dogs will you have to sell to break even? The y-term will be your overall total and the x will be the number of burgers/dogs sold. Make a quick graph of this equation on the provided blank graph to the right.

burgers = \$360
1 burger = \$90
Profit = 100 - 90 = \$10



income = profit - loss

$$y = \text{profit} - \text{loss}$$

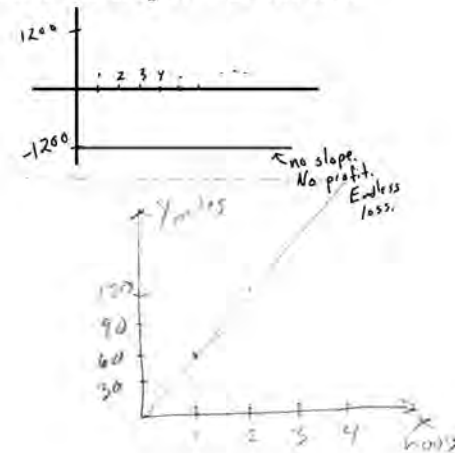
$$y = \$10x - \$1200$$

$m = 10$
 $b = -1200$

2.2b After 2 baseball games you realize that this business is not going well. You have not sold a single hamburger or hotdog so you lower the price of your products to \$90. How many sales will it take to break even at this new price? Write an equation for this situation. What is the slope? Put this new equation on the same graph for part 2.2a.

Problem is, at \$90 there is no profit $\rightarrow m = 0$

If $m = 0$ then $y = -1200$



2.3 You are in a van headed for a Grateful Dead concert in Cleveland. Your driver is going 60 miles per hour. Assuming you are able to continue going 60mph, write an equation that will output how far you have travelled as a function of time? The y-term will be the total distance travelled. The x-term will be hours traveled. The slope will be 60mph. Sketch a quick graph to the right.

$$y = 60x$$

where $x = \text{hours}$

$m = 60$
 $b = 0$

