BLC150 – HomeWork 5 Name:________________________

The Loanshark

You've been lent $100 from the *Nostra Res Bardi.*
They are charging you an interest rate of 10%, compounded weekly.

1st Week: $A_1 = \$100 + 10\%(\$100) = \$110$ $A_1 = \$100 + (0.1)(\$100) = \$110$ $A_1 = \$100(1 + 0.1) = \$110$ $A_1 = \$100(1.1) = \$110$ $A_1 = \$110$	A_1 means "the Amount accrued after 1 week." $100 is the Principal, the initial amount. 10%($100) is the interest added onto the Principal. To be usable mathematics, we'll write it, "(0.1)($100)." Notice that we can factor out $100 and simplify the equation.
2nd Week: $A_2 = \$110 + (0.1)(\$110) = \$121$ $A_2 = \$110(1 + 0.1) = \$121$ $A_2 = \$110(1.1) = \$121$	After 2 weeks another 10% is added. But now it is 10% of $110.
3rd Week: $A_3 = \$121 + (0.1)(\$121) = \$133.10$ $A_3 = \$121(1 + 0.1) = \$133.10$ $A_3 = \$121(1.1) = \$133.10$	After 3 weeks another 10% is added. But now it is 10% of $121. You get the idea.
$A_3 = \$121(1.1) = \$133.10$ $A_3 = A_2(1.1) = \$133.10$	Now, let's look at these three weeks of accrued interest, the equation for A_3. Notice from above, that A_2 = $121. So substitute A_2 in for $121.
$A_3 = \$110(1.1)(1.1) = \$133.10$	Now notice from above that A_2 can also be written as $110(1.1). So substitute this in as well.
$A_3 = \$100(1.1)(1.1)(1.1) = \$133.10$	But $110 is just A_1, which is $100(1.1)
$A_3 = \$100(1.1)^3 = \$133.10$	Are you seeing the pattern? Three weeks of accrued interest and three 1.1s. We could write these as $(1.1)^3$.
$A_3 = \$100(1.1)^4 = \$146.41$	To find the 4th week total you just make the exponent a 4.

Eq. 1 $$A_t = P\left(1+\frac{r}{n}\right)^{nt}$$	Generalize. Where *A* is the final amount, *P* is the principal, *r* is the interest rate per time period, *n* is the number of times the interest rate is compounded per time period, and *t* is the amount of time in total. Warning: All time measurements should be in the same units.
$A_4 = \$100\left(1+\frac{0.1}{1}\right)^4$ $= \$146.41$	Let's see if this formula gives us the correct results. $P = \$100$ $r = 10\%$ per week, written 0.1 $n = 1$ (it is only compounded weekly) $t = 4$ weeks
$A(4) = \$100\left(1+\frac{0.1}{7}\right)^{28}$ $A(4) \cong \$100(1.0143)^{28}$ $A(4) \cong \$148.76$	Here is the same problem but with the interest compounded daily, seven times per week, $n = 7$. As you can see, there are now 28 compounding periods (see exponent) but the interest rate is significantly reduced from 1.1 to approx. 1.0143. And look! The amount got a little bigger from the example immediately above. Compounded weekly the total was $146.41. Compounded daily it increased to $148.76. Guess who likes lots of compounding interest periods –lenders or borrowers?
$\$148.76 - \$146.41 = \$2.35$ $\$146.41x = \$2.35$ $x \cong 0.016 = 1.6\%$	That's about a 1.6% increase.

Exercise 1: USING EQ. 1. $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$

1. Now you get to be the banker. Find the total amount you will have if you invest $100, at 8% (annual rate), compounded **annually** for …

You will want to use a calculator for this. Ask me in class how to do exponents if you are unsure.

E.g.) *t = 1 year; P = $100; r = 8% = 0.08; n = 1* (because it is compounded annually) $A(t) = P\left(1+\frac{r}{n}\right)^{nt} = \$100\left(1+\frac{0.08}{1}\right)^{1(1)} = \$100(1.08) = \$\mathbf{108}.$ After one year, you will have 108%.
a) *t = 10 years; P = $100; r = 0.8; n = 1* (because it is compounded annually) $A(10) = \$100\left(1+\frac{0.08}{1}\right)^{10(1)} =$

b) *t = 30 years;* *P = $100;* *r = 0.8;* *n = 1* (because it is compounded annually) $A(30) = \$100\left(1+\frac{0.08}{1}\right)^{30(1)} =$

2. Find the total amount you will have if you invest $100, at 8% (annual rate), compounded **seasonally**.
That means *n = 4*.... 4 seasons in a year. I did part b to show you how.

a) *t = 1 year*
b) *t = 10 years* $A(10) = \$100\left(1+\frac{0.08}{4}\right)^{4\cdot 10} = \$100(1.02)^{40} \cong \$220.80.$ You more than doubled your money in 10 years.
c) *t = 30 years*

3. Find the total amount you will have if you invest $100, at 8% (annual rate), compounded **daily**.
That means n = 365 or if you want to be fancy, 365.25.

a) *t = 1 year*
b) *t = 10 years*
c) *t = 30 years*

4. Now you have to figure it all out. You lend $400 to an acquaintance. You charge her 7% per month, compounded monthly. After 2 years, how much does she owe you?

5. a. You borrow \$15,000 from your Visa card in order to pay for fees, expenses, and tuition at school. The bank charges you 17% annual interest, compounded annually. How much do you owe the bank after 5 years? Hint: What is P? What is r? What is n? What is t?

b. Same set-up as above, P = \$15,000, r = 0.17 (annual interest rate), but instead of being compounded annually, it is compounded daily. How much do you owe the bank after 5 years. (assume that a year is 365.25 days long).

6. Here is a new formula: $T(t) = 2.72^{0.2t}$.

Fill in the table below with your results and then plot all of the points. Your answers can be rounded to the nearest 100th. To plot the first example, your point is (0, 1). The second example is (4, 2.23). The x is in the parentheses after the T (it is your input) and the y is what T equals. After you have plotted all of the points, sketch a curve to fit the points.

E.g.	$T(\mathbf{0}) = 2.72^{0.2\cdot 0} = 2.72^{0} =$	**1**
1	$T(1) = 2.72^{0.2\cdot 1} = 2.72^{0.2} =$	
2	$T(2) = 2.72^{0.2\cdot 2} =$	
3	$T(3) = 2.72^{0.2\cdot 3} =$	
E.g.	$T(\mathbf{4}) = 2.72^{0.2\cdot 4} =$	**2.23**
4	$T(5) =$	
5	$T(7) =$	
6	$T(15) =$