

Homework 4: I suggest you do the homework on scrap paper and then transfer your process and answers to this homework sheet. In other words, do a rough draft and then a final draft on this paper. I'm going to start grading for presentation. Treat these as short papers. I recommend pencil over pen, for obvious reasons.

Epimetheus [Afterthought]: Problem problems I saw after grading HW-2.

a) What three consecutive odd numbers add up to 339.

$$x + x + 2 + x + 4 = 339$$

b) $\frac{\sqrt{x+4}}{5} + 12 = 13$

$$\frac{\sqrt{x+4}}{5} = 1 \Rightarrow \sqrt{x+4} = 5$$

$$x+4 = 25$$

$$x = 21$$

c) $\frac{x^2-1}{2} = 12 \Rightarrow x^2-1 = 24 \Rightarrow x^2 = 25$

$$x = \pm 5$$

d) Solve to satisfy both: $x + y = 5$ and $3x - 2y = 5$

$$x = 5 - y$$

$$3(5 - y) - 2y = 5$$

$$15 - 3y - 2y = 5$$

$$15 - 5y = 5$$

$$10 = 5y$$

$$y = 2$$

$$x = 5 - 2$$

$$x = 3$$

PERCENTAGES

Percent quite literally means *per-centum*, which means per-100.

A percentage is actually a fraction. 50% should be written $\frac{50}{100}$, fifty per one hundred.

The symbol, %, sort of implies this. Perhaps the symbol should really be $\frac{1}{100}$, instead of %. That way

25% would be written $25(\frac{1}{100}) = \frac{25}{100}$. That way you would never need to convert percentage to

decimal. It would be obvious. You've got 25 hundredths.

50% is the same as $\frac{50}{100} = \frac{1}{2} = 0.5 = 0.50 = \frac{5}{10} \dots etc.$

Most of the time 0.5 or 1/2 is how you actually use 50% in any mathematical equation.

We may think in percents, but we do math in *per-100s*.

1. Convert the following to a fraction, then to a decimal, then solve. (Use a calculator when needed.)

	E.g. 50% of 30.	a. 50% of 70.	b. 25% of 200.
Fraction version	$\frac{50}{100} \cdot 30 = \frac{150}{100} = 15$ or $\frac{1}{2} \cdot 30 = \frac{30}{2} = 15$	$\frac{50}{100}(70) = \frac{1}{2}(70) = 35$	$\frac{25}{100}(200) = \frac{1}{4}(200) = 50$
Decimal version	$= 0.5 \cdot 30 = 15$	$0.5(70) = 35$	$0.25(200) = 50$

	c. 15% of 120.	d. 5% of 1000.	e. 8% of 200.
Fraction version	$\frac{15}{100} \cdot \frac{120}{1} = \frac{15 \cdot 12}{10} = 18$	$\frac{5}{100}(1000) = \frac{5000}{100} = 50$	$\frac{8}{100}(200) = 8(\frac{2}{10}) = 16$
Decimal version	$0.15(120) = 18$	$0.05 \cdot 1000 = 50$	$.08(200) = 16$

Useful tricks: 10% of 134 is 13.4. 20% of 134 is just 13.4 times 2 which is 26.8.

1% of 122 is 1.22. 2% is 2.44. 4% is double again: 4.88.

Just move the decimal point....and double it or triple or quadruple it.

2. You are at a nice restaurant and you are paying the bill. The service was good but not out of the ordinary so you want to tip 20%. The bill is \$132. What should you leave as a tip? [Hint: What is 10%? Just double it.]

$\frac{20}{100} (\$132) = ?$ $10\% \text{ of } \$132 = \$13.2 \rightarrow \text{times 2 will be } 20\% = \26.40

3. You are at a nice restaurant, but the service is pretty bad. You never got water and the silverware was a bit gross. You want to leave a 15% tip. The bill was \$160. What's the tip? [Hint: What is 10%? Now what is 5%?... It's just half of 10%. Add them together and you'll get 15%.]

$\frac{15}{100} (\$160) = ?$ $10\% \text{ is } 16$
 $5\% \text{ is } 8 \rightarrow \text{add them} \rightarrow \24

4. Amazing service. Amazing meal. Free desert. What is 30% of \$220. [Hint: It's just 10% of \$220 times 3.]

$\frac{30}{100} (\$220) = ?$ $10\% \text{ is } 22$
 $20\% \text{ is } 44$ $30\% \text{ is } \$66.00$

5. Guess the answers to the following. Don't use a calculator. Just circle the answer.

a. 2% of 100.	a. 4	b. 2	Note: What's 1%. Now double it.	b. 2
	c. 0	d. 80		

b. 10% of 100.	a. 5	b. 10	What's one tenth of 100?	b. 10
	c. 15	d. 20		

c. 12% of 100.	a. 120	b. 12	What's 10% of 100 plus 2% of 100?	b. 12
	c. 1.2	d. 8		

d. 12% of 200	a. 240	b. 120	Now double the last one since the amount doubled.	d. 24
	c. 12	d. 24		

e. 12% of 500	a. 60	b. 36	Now multiply the answer from c. by 5.	a. 60
	c. 78	d. 250		

f. 25% of 400.	a. 50	b. 100	What's 1/4 of 100, multiplied by 4.	b. 100
	c. 200	d. 88		

g. 15% of 300.	a. 40	b. 45	What's 10% of 300. What's half of that. Add them together to get 15%.	b. 45
	c. 65	d. 100		

h. 30% of 300.	a. 30	b. 60	What's 30% of 100. Multiply that by 3 for 30% of 300.	c. 90
	c. 90	d. 120		

i. 0.5% of 200	a. 0	b. 1	What's 1% of 200. Halve it.	b. 1
	c. 2	d. 4		

j. 2.5% of 500	a. 5	b. 12.5	What's 1% of 500. Multiply that by 2.5 (or 2 + 1/2)	b. 12.5
	c. 25	d. 50		

6. Set up an equation and then solve it.

E.g. If a shirt is 20% off and it originally cost \$40. What does it now cost?

$$\$40 - (0.20)(\$40) = \$40 - \$8 = \$32.$$

a. A car costs \$22,500. The dealer says he'll give you 12% (0.12) off. How much is the car now?

$$(\$22,500)(0.12) = \$2700$$

$$\$22,500 - \$2700 = \$19,800$$

b. There was a 10% increase in homicide in Metropolis from 2016 to 2017. There were 120 homicides in 2016. How many were there in 2017.

$$10\% \text{ of } 120 = 12$$

$$\rightarrow 120 + 12 = 132$$

c. You need to buy 3 shirts for work. It's a "Buy 2, Get 1 Free" sale at Barney's. You could buy 2 shirts and get a third for free and pay \$20. The store next door is having a 33 $\frac{1}{3}$ % off sale. Their shirts before the discount cost \$10. Which sale is better?

Buy 2, get 1 free
means \$20 for 3 shirts
 $\frac{20}{3} \approx \$6.67$ per shirt

SAME

33 $\frac{1}{3}$ % (\$10) is $0.\bar{3}(\$10) = \3.33 off

One shirt costs $\$10 - 3.33 = \6.67

Note $0.\bar{3} = 0.3333\dots$

d. Your friend is a hopeless slacker. He complains that his supplier of Dorito's Cool Ranch™ just increased prices by 15%. He used to pay \$40 per carton. What does he have to pay now?

$$\$40 + 0.15(40) = \$46$$

e. The population of Annandale was 15,230 in 2016. In 2017 the population was 14,250. How much did the population decrease (in percent) from 2016 to 2017.

$15,230 - 15,230x = 14,250$. Solve for x and convert to percentage.

$$15,230x = 980 \rightarrow x = \frac{980}{15,230} \approx 0.064 \sim 6\% \text{ decrease}$$

f. Your friend complains that his favorite study aid, NoDoz™, used to contain 200mg of caffeine. Now it contains only 165mg. How much did it decrease in terms of percentage?

$$200 - 200(x) = 165$$

$$200x = 35$$

$$x = \frac{35}{200} = 0.175 = 17.5\%$$

g. Your friend tells you that his IQ is 16% higher than the average Mauritanian. You are weirdly surprised how good your friend is at math considering his continuous use of NoDoz™ and you are also surprised by his knowledge of strange trivia. What is his IQ? Are you impressed? Do you trust IQ statistics? [You'll have to look up the average IQ of a Mauritanian.]

Average IQ of Mauritanian = 76

$$\text{Your friend's IQ} = 76 + 76(0.16) = 88.16$$

Not Impressed

Don't trust I.Q.

7. And now for a bunch of fractions. Solve for the variable and show your work.

<p>a. $\frac{x}{3} + \frac{x}{2} = \frac{5}{6}$</p> $\frac{2x}{6} + \frac{3x}{6} = \frac{5x}{6} \} \textcircled{x=1}$	<p>b. $\frac{x}{4} + \frac{x}{3} = \frac{7}{6} \rightarrow \frac{4x}{12} + \frac{3x}{12} = \frac{7}{6} \rightarrow \frac{7x}{12} = \frac{7}{6}$</p> $\rightarrow \frac{12(7x)}{12} = \frac{12(7)}{6} \rightarrow 7x = 14 \rightarrow \textcircled{x=2}$
<p>b. $\frac{1}{x} + \frac{8}{3} = 3$</p> $\frac{1}{x} = 3 - \frac{8}{3} = \frac{9}{3} - \frac{8}{3}$ $\frac{1}{x} = \frac{1}{3} \} \textcircled{x=3}$	<p>c. $\frac{x}{4} + \frac{3x}{12} = 2$</p> $\frac{3x}{12} + \frac{3x}{12} = \frac{6x}{12} = \frac{x}{2} = 2$ $\textcircled{x=4}$
<p>d. $\frac{1}{3x} + \frac{3}{x} = \frac{2}{3}$ [Hint: you'll eventually want to cross-multiply.]</p> $\frac{1}{3x} + \frac{9}{3x} = \frac{2}{3}$ $\frac{10}{3x} = \frac{2}{3} \Rightarrow 6x = 30$ $\textcircled{x=5}$	<p>e. $\frac{36}{2x-3} = \frac{4}{1}$ [Hint: Start out by cross-multiplying.]</p> $8x - 12 = 36$ $8x = 48$ $\textcircled{x=6}$
<p>f. $\frac{22}{2x-3} = 2$</p> <p>Cross Multiply.</p> $4x - 6 = 22$ $4x = 28$ $\frac{4x}{4} = \frac{28}{4}$ $\textcircled{x=7}$	<p>g. $\frac{8}{x^2} + \frac{2}{x} = \frac{3}{x}$</p> $\frac{8}{x^2} + \frac{2x}{x^2} = \frac{3x}{x^2}$ $\frac{8}{x^2} = \frac{x}{x^2} \text{ multiply both sides by } x^2$ $\textcircled{8=x}$
<p>h. $(x-9)^2 = 0$ [Hint: Just think about it. Don't do any "algebra".]</p> $\textcircled{x=9}$	<p>i. $x-11 = -1$ [Hint: Just think about it. Don't do any "algebra".]</p> $\textcircled{x=10}$