

# BLC150–Algebra Workshop

## Homework 3

Name: \_\_\_\_\_

Add in the missing prime numbers in this sequence...

2	3	5		11	13	17		23	29	31	37		43	47	53	59		67
	73	79	83	89	97		103	107	109	113	127	131	137	139	149	151	157	163
167	173	179	181	191	193	197	199	211	223	227	229	233	239	241	251	257	263	269

Trivia: Choose any prime number on this list. Double it, and there will be a prime either +1 or -1 from the doubled prime you chose. E.g.  $13(2) = 26$ .

Now, scribble in every prime number on this grid of numbers. I filled in the last 2 to show you what I mean.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127	128	129
130	131	132	133	134	135	136	137	138	139
140	141	142	143	144	145	146	147	148	149
150	151	152	153	154	155	156	157	158	159
160	161	162	163	164	165	166	167	168	169
170	171	172	173	174	175	176	177	178	179
180	181	182	183	184	185	186	187	188	189
190	191	192	193	194	195	196	197	198	199
200	201	202	203	204	205	206	207	208	209
210	211	212	213	214	215	216	217	218	219
220	221	222	223	224	225	226	227	228	229
230	231	232	233	234	235	236	237	238	239
240	241	242	243	244	245	246	247	248	249
250	251	252	253	254	255	256	257	258	259
260	261	262	263	264	265	266	267	268	269
270	271	272	273	274	275	276	277	278	279

Do you see a pattern? Analyze this array of numbers and make 2 or 3 observations to the right of the grid.

Now lets do the Sieve of Eratosthenes (d. 194 BC)

1. Start with 2, the first prime. Cross out every second number on the list after 2. [Don't cross out 2. It's a prime.]  
Cross out all multiples of 2, because they are all obviously not prime, since they can be factored by 2.  
E.g. Cross out 4, 6, 8, etc.
2. Find the next number that is not crossed out. That would be 3. It's a prime. Cross out every 3rd number after 3.  
E.g. 6, 9, 12, etc. (Some of these were already crossed out.)
3. Find the next number that is not crossed out. That should be a 5. Cross every fifth number after 5.  
E.g. 10, 15, 20, etc.
4. Do this until you can't. What remains are the primes.

		2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39

Trivia:

How to find even perfect numbers (described by Euclid).

A **perfect number** is a positive integer that is equal to the sum of its proper positive divisors.  
 6 can be divided evenly by 1, 2, and 3. A perfect number has the following quality:  $1 + 2 + 3 = 6$   
 8 is not perfect. Divisors: 1, 2, 4.  $1 + 2 + 4 \neq 8$   
 28 is perfect: Divisors: 1, 2, 4, 7, 14.  $1 + 2 + 4 + 7 + 14 = 28$ .

Euclid describes the following relationship between prime numbers and perfect numbers:

$$2^{p-1}(2^p - 1), \text{ for } p \text{ being a prime number,}$$

For prime=2	$2^1(2^2 - 1) = 6 = 1 + 2 + 3 = 6$
For prime = 3	$2^2(2^3 - 1) = 28 = 1 + 2 + 4 + 7 + 14 = 28$
For prime = 5	$2^4(2^5 - 1) = 496$
For prime = 7	$2^6(2^7 - 1) = 8128$
For prime = 11	You figure it out.

1. Break the following numbers down into their prime factors as shown in the examples. On the right side of the answer boxes I also wrote the answers in exponential form when it was appropriate. If you know how to do this great, but if you don't, don't worry about it just yet.

a.	15 =	= 3 · 5
b.	152 =	= 2 · 76 = 2 · 2 · 38 = 2 · 2 · 2 · 19 <span style="float: right;">= (2<sup>3</sup>) · 19</span>
c.	36 =	
d.	18 =	
e.	250 =	= 5 · 50 = 5 · 5 · 10 = 5 · 5 · 5 · 2 <span style="float: right;">= 2 · 5<sup>3</sup></span>
f.	66 =	
g.	23 =	= 23 That's it. Twenty-three is a prime number.
h.	93 =	
i.	49 =	
j.	82 =	
k.	161 =	= 23 · 7
l.	133 =	

2. The Dreaded Fraction: Convert the following fractions by filling in the missing numbers. **Members of each row are all equal.** You can check your answers with a calculator if you want.

1.	$\frac{1}{2} =$	$= \frac{2}{4} =$	$= \frac{4}{8} =$	$= \frac{8}{16}$
2.	$\frac{1}{3}$	$\frac{\quad}{6}$	$\frac{\quad}{12}$	$\frac{8}{24}$
3.	$\frac{2}{2}$	$\frac{\quad}{4}$	$\frac{8}{8}$	$\frac{\quad}{16}$
4.	$\frac{1}{4}$	$\frac{\quad}{8}$	$\frac{\quad}{16}$	$\frac{\quad}{20}$
5.	$\frac{2}{1}$	$\frac{4}{2}$	$\frac{8}{\quad}$	$\frac{\quad}{6}$
6.	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{8}{\quad}$	$\frac{\quad}{21}$
7.	$\frac{4}{5}$	$\frac{40}{\quad}$	$\frac{\quad}{500}$	$\frac{\quad}{20}$
8.	$\frac{5}{100}$	$\frac{1}{20}$	$\frac{15}{\quad}$	$\frac{\quad}{40}$
9.	$\frac{2}{1/2}$	$\frac{4}{1}$	$\frac{8}{2}$	$\frac{16}{4}$

10.	$\frac{3}{1/2}$	$\frac{\quad}{1}$	$\frac{\quad}{2}$	$\frac{24}{4}$
11.	$\frac{6}{1/3}$	$\frac{12}{2/3}$	$\frac{18}{1}$	$\frac{\quad}{2}$
12.	$\frac{8}{1}$	$\frac{4}{\quad}$	$\frac{16}{\quad}$	$\frac{80}{\quad}$
13.	$\frac{1/2}{2}$	$\frac{1}{4}$	$\frac{2}{\quad}$	$\frac{8}{\quad}$
14.	$\frac{-1}{3}$	$\frac{-2}{6}$	$\frac{2}{-6}$	$\frac{\quad}{12}$
15.	$\frac{-2}{2}$	$\frac{-1}{\quad}$	$\frac{-6}{6}$	$\frac{\quad}{8}$
16.	$\frac{8}{16}$	$\frac{4}{\quad}$	$\frac{\quad}{4}$	$\frac{1}{\quad}$
17.	$\frac{3}{16}$	$\frac{\quad}{32}$	$\frac{\quad}{8}$	$\frac{\quad}{4}$
18.	$\frac{8\pi}{16}$	$\frac{4\pi}{\quad}$	$\frac{\quad}{4}$	$\frac{\pi}{\quad}$

3. Evaluate these. You need to find common denominators. I did the first two. **Each answer is larger than the previous answer.** Write all answers as fractions, not mixed fractions. I.e.  $\frac{4}{3}$  not  $4\frac{1}{3}$ .

a. $\frac{1}{2} + \frac{2}{4} = \frac{1}{2} + \frac{1}{2} = 1$	b. $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
c. $\frac{3}{2} + \frac{1}{4} =$	d. $\frac{3}{2} + \frac{2}{4} =$
e. $1 + \frac{3}{4} + \frac{3}{8} =$	f. $1 + \frac{5}{4} + \frac{3}{8} =$

g. $\frac{3}{5} + \frac{7}{3} =$	h. $\frac{3}{5} + \frac{7}{2} =$
i. $\frac{25}{5} - \frac{1}{2} =$	j. $\frac{23}{4} - \frac{2}{3} =$
k. $6 - \frac{1}{10} =$	l. $\frac{100}{5} - \frac{10}{2.5} =$
m. $\frac{5}{1/2} =$	n. $\frac{6}{1/2} =$
o. $\frac{2}{1/7} =$	p. $\frac{3}{1/7} =$
q. $\frac{46/4}{1/2} =$	r. $\frac{300/2 + 500/10}{2} =$