

BLC150–Algebra Workshop Homework 3

Name: *Daniel's Copy*

Add in the missing prime numbers in this sequence...

2	3	5	(7)	11	13	17	(19)	23	29	31	37	(41)	43	47	53	59	(61)	67
(71)	73	79	83	89	97	(101)	103	107	109	113	127	131	137	139	149	151	157	163
167	173	179	181	191	193	197	199	211	223	227	229	233	239	241	251	257	263	269

Trivia: Choose any prime number on this list. Double it, and there will be a prime either +1 or -1 from the doubled prime you chose. E.g. $13(2) = 26$.

Now, scribble in every prime number on this grid of numbers. I filled in the last 2 to show you what I mean.

	0	14	1	15	0	1	0	15	0	13									
4	0	1	2	3	4	5	6	7	8	9									
4	10	11	12	13	14	15	16	17	18	19									
2	20	21	22	23	24	25	26	27	28	29									
2	30	31	32	33	34	35	36	37	38	39									
3	40	41	42	43	44	45	46	47	48	49									
2	50	51	52	53	54	55	56	57	58	59									
2	60	61	62	63	64	65	66	67	68	69									
3	70	71	72	73	74	75	76	77	78	79									
2	80	81	82	83	84	85	86	87	88	89									
(1)	90	91	92	93	94	95	96	97	98	99									
4	100	101	102	103	104	105	106	107	108	109									
(1)	110	111	112	113	114	115	116	117	118	119									
(1)	120	121	122	123	124	125	126	127	128	129									
3	130	131	132	133	134	135	136	137	138	139									
(1)	140	141	142	143	144	145	146	147	148	149									
2	150	151	152	153	154	155	156	157	158	159									
2	160	161	162	163	164	165	166	167	168	169									
2	170	171	172	173	174	175	176	177	178	179									
(1)	180	181	182	183	184	185	186	187	188	189									
4	190	191	192	193	194	195	196	197	198	199									
→ 0	200	201	202	203	204	205	206	207	208	209									
(1)	210	211	212	213	214	215	216	217	218	219									
3	220	221	222	223	224	225	226	227	228	229									
2	230	231	232	233	234	235	236	237	238	239									
(1)	240	241	242	243	244	245	246	247	248	249									
2	250	251	252	253	254	255	256	257	258	259									
2	260	261	262	263	264	265	266	267	268	269									
2	270	271	272	273	274	275	276	277	278	279									

There are 59 primes on this table.

About 2.1 primes per row.

Only one even prime → 2

25 primes in 0-100

21 primes in 100-200

The 3s & 7s columns are the densest

Do you see a pattern? Analyze the array & make observations.

Now lets do the Sieve of Eratosthenes (d. 194 BC)

1. Start with 2, the first prime. Cross out every second number on the list after 2. [Don't cross out 2. It's a prime.]
Cross out all multiples of 2, because they are all obviously not prime, since they can be factored by 2.
E.g. Cross out 4, 6, 8, etc. [Shown in gray here.]
2. Find the next number that is not crossed out. That would be 3. It's a prime. Cross out every 3rd number after 3.
E.g. 6, 9, 12, etc. (Some of these were already crossed out.) [Shown in pink.]
3. Find the next number that is not crossed out. That should be a 5. Cross every fifth number after 5.
E.g. 10, 15, 20, etc. [Shown in green.]
4. Do this until you can't. What remains are the primes.

		2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39

Trivia:

How to find even perfect numbers (described by Euclid).

A **perfect number** is a positive integer that is equal to the sum of its proper positive divisors.
6 can be divided evenly by 1, 2, and 3. A perfect number has the following quality: $1 + 2 + 3 = 6$
8 is not perfect. Divisors: 1, 2, 4. $1 + 2 + 4 \neq 8$
28 is perfect: Divisors: 1, 2, 4, 7, 14. $1 + 2 + 4 + 7 + 14 = 28$.

Euclid describes the following relationship between prime numbers and perfect numbers:
 $2^{p-1}(2^p - 1)$, for p being a prime number which fits the condition, $2^p - 1$ is a prime number,

For prime = 2	$2^1(2^2 - 1) = 6 = 1 + 2 + 3 = 6$
For prime = 3	$2^2(2^3 - 1) = 28 = 1 + 2 + 4 + 7 + 14 = 28$
For prime = 5	$2^4(2^5 - 1) = 496 = 1+2+4+8+16+31+62+124+248$
For prime = 7	$2^6(2^7 - 1) = 8128 = 1+2+4+8+16+32+64+127+254+508+1016+2032+4064$
For prime = 13	You figure it out. = 33,550,336 = yikes.

1. Break the following numbers down into their prime factors as shown in the examples. On the right side of the answer boxes I also wrote the answers in exponential form when it was appropriate. If you know how to do this great, but if you don't, don't worry about it just yet.

a.	15 =	= 3 · 5	
b.	152 =	= 2 · 76 = 2 · 2 · 38 = 2 · 2 · 2 · 19	= (2 ³) · 19
c.	36 =	= 2 · 18 = 2 · 2 · 9 = 2 · 2 · 3 · 3 Or... 6 · 6 = 2 · 3 · 2 · 3...	= 2 ² · 3 ²
d.	18 =	= 2 · 3 · 3 = 2 ² · 3	= 2 ² · 3
e.	250 =	= 5 · 50 = 5 · 5 · 10 = 5 · 5 · 5 · 2	= 2 · 5 ³
f.	66 =	= 6 · 11 = 2 · 3 · 11	
g.	23 =	= 23 That's it. Twenty-three is a prime number.	
h.	93 =	= 3 · 31	
i.	49 =	= 7 ²	= 7 ²
j.	82 =	= 2 · 41	
k.	161 =	= 23 · 7	
l.	133 =		= 7 · 19

2. The Dreaded Fraction: Convert the following fractions by filling in the missing numbers. **Members of each row are all equal.** You can check your answers with a calculator if you want.

1.	$\frac{1}{2} =$	$= \frac{2}{4} =$	$= \frac{4}{8} =$	$= \frac{8}{16}$
2.	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{4}{12}$	$\frac{8}{24}$
3.	$\frac{2}{2}$	$\frac{4}{4}$	$\frac{8}{8}$	$\frac{16}{16}$
4.	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{4}{16}$	$\frac{5}{20}$
5.	$\frac{2}{1}$	$\frac{4}{2}$	$\frac{8}{4}$	$\frac{12}{6}$
6.	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{8}{12}$	$\frac{14}{21}$
7.	$\frac{4}{5}$	$\frac{40}{50}$	$\frac{400}{500}$	$\frac{16}{20}$
8.	$\frac{5}{100}$	$\frac{1}{20}$	$\frac{15}{300}$	$\frac{2}{40}$
9.	$\frac{2}{1/2}$	$\frac{4}{1}$	$\frac{8}{2}$	$\frac{16}{4}$

10.	$\frac{3}{1/2}$	$\frac{6}{1}$	$\frac{12}{2}$	$\frac{24}{4}$
11.	$\frac{6}{1/3}$	$\frac{12}{2/3}$	$\frac{18}{1}$	$\frac{36}{2}$
12.	$\frac{8}{1}$	$\frac{4}{1/2}$	$\frac{16}{2}$	$\frac{80}{10}$
13.	$\frac{1/2}{2}$	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{8}{32}$
14.	$\frac{-1}{3}$	$\frac{-2}{6}$	$\frac{2}{-6}$	$\frac{-4}{12}$
15.	$\frac{-2}{2}$	$\frac{-1}{1}$	$\frac{-6}{6}$	$\frac{-8}{8}$
16.	$\frac{8}{16}$	$\frac{4}{8}$	$\frac{2}{4}$	$\frac{1}{2}$
17.	$\frac{3}{16}$	$\frac{6}{32}$	$\frac{15}{8}$	$\frac{0.75}{4}$
18.	$\frac{8\pi}{16}$	$\frac{4\pi}{8}$	$\frac{2\pi}{4}$	$\frac{\pi}{2}$

3. Evaluate these. You need to find common denominators. I did the first two. **Each answer is larger than the previous answer.** Write all answers as fractions, not mixed fractions. I.e. $\frac{4}{3}$ not $4\frac{1}{3}$.

a. $\frac{1}{2} + \frac{2}{4} = \frac{1}{2} + \frac{1}{2} = 1$	b. $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
c. $\frac{3}{2} + \frac{1}{4} = \frac{7}{4}$	d. $\frac{3}{2} + \frac{2}{4} = \frac{8}{4}$
e. $1 + \frac{3}{4} + \frac{3}{8} = \frac{17}{8}$	f. $1 + \frac{5}{4} + \frac{3}{8} = \frac{21}{8}$

g. $\frac{3}{5} + \frac{7}{3} = \frac{44}{15}$	h. $\frac{3}{5} + \frac{7}{2} = \frac{41}{10}$
i. $\frac{25}{5} - \frac{1}{2} = 4.5$	j. $\frac{23}{4} - \frac{2}{3} = \frac{61}{12}$
k. $6 - \frac{1}{10} = \frac{59}{10}$	l. $\frac{100}{5} - \frac{10}{2.5} = \frac{80}{5}$
m. $\frac{5}{1/2} = 10$	n. $\frac{6}{1/2} = 12$
o. $\frac{2}{1/7} = 14$	p. $\frac{3}{1/7} = 21$
q. $\frac{46/4}{1/2} = 23$	r. $\frac{300/2 + 500/10}{2} = 100$