CLASS 1
BLC-150: THE ALGEBRA WORKSHOP

A prime number is a natural number (counting number or positive integer) **greater than 1 that cannot be formed by multiplying two smaller natural numbers.** Put another way, a prime number is a natural number that has exactly two distinct natural number factors (or divisors), namely 1 and the number itself.

A natural number greater than 1 that is not prime is called a composite number.

Is 2 a prime number? It's "greater than 1." Yes. But can it be made "by multiplying two smaller natural numbers"?
No. There are no natural numbers smaller than 2 which can be multiplied together to make 2. The only posibility of a natural number less than 2 is 1, and there is no way to multiply 1 by 1 to get 2. Therefore, 2 is prime.
Put another way, the only natural number factors of 2 are 1 and 2, ... 1 and itself. Therefore 2 is prime.

How to find prime numbers?
One way is the Sieve of Eratosthenes (d. 194 BC)

a. Start with 2, the first prime. Cross out every number that is a multiple of 2 on this list. That will be every even number greater than 2. For example, you will cross out 26. Why? Because it is even, and because it is a composite number. It can be expressed as 2 x 13. Therefore 26 is not a prime number. A prime number cannot be expressed as a product of two lesser natural numbers.

b. Go to the next number that is not crossed out. That would be 3. Is it a prime number? Can it be expressed as a product of two natural numbers less than 3? The only natural number candidates are 1 and 2. Can you multiply 1 x 1, 1 x 2, or 2 x 2 and get 3? No. Therefore 3 is prime. Its only natural number factors are 1 and 3. Now cross out every 3rd number after 3. E.g. Cross out 6, 9, 12, 15, etc.

c. Find the next number that is not crossed out. That should be a 5. [4 should be crossed out from step a.] Is 5 prime? Are there any factors of 5 that are natural numbers less than 5? If so, cross out every fifth number after 5. E.g. 10, 15, 20, etc.

d. Go to then next number that is not crossed out and repeat step c. Do this until you can't anymore.

What remains are the primes up to 100.

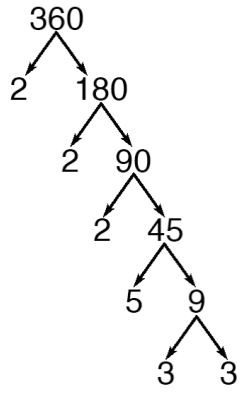
~~1~~	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Now, choose a number that is crossed off from the table above.
How about 77.

What are its prime factors? What numbers lower than 77 can be multiplied together to get 77?
Answer: 11 and 7. These are the prime factors of 77.

How about 42? What are its prime factors?
Well.... 2 and 21 multiplied together give 42. 2 and 21 are factors, but are they the prime factors?
One of them is. 2, but 21 is not prime, so you need to keep going...
42 = 2 x 21 = 2 x 3 x 7. This is the **prime factorization** of 42.

Larger numbers can get trickier.
The standard method for prime factorization is graphical, so I'll just present this the picture.



The prime factorization of 360 is
$2 \cdot 2 \cdot 2 \cdot 5 \cdot 3 \cdot 3$

or in order...
$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

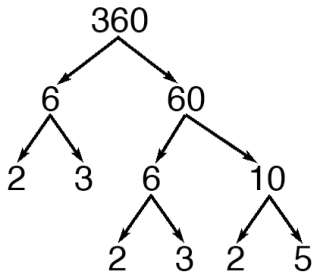
or fancy-style...
$2^3 \cdot 3^2 \cdot 5$

There is more than one way to skin a cat...

But you end up with the same prime factorization.
$2 \cdot 3 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

or fancy-style...

$2^3 \cdot 3^2 \cdot 5$



Go to homework assignment for Class 2. [HW-2]