Name:

BLC150-Homework 14 - Factoring and the Quadratic Formula.

Example 1: $x^2 + 2x = 0$. What values of x will make this true? It's a little hard to see it. But if you rewrite it in a different way, it becomes much clearer.

Factor out x from the left side of this equation: $x^2 + 2x = 0$.

$$x(x + 2) = 0$$

What values of x will make this work? Obviously 0 and -2. x = 0 and -2

Example 2: $4x^2 + 8x = 0$

Factor out a 4x .... $4x(x + 2) = 0$

Clearly if x = 0 or -2 this equation will work out.

If you don't see it clearly, you could also divide both sides by 4 to get...

$$\frac{4x(x+2)}{4} = \frac{0}{4} \quad \rightarrow \quad x(x+2) = 0 \quad \rightarrow \quad x = 0 \; and \; -2$$

Example 3: $4x^2 - 2x = 0$

Method 1: $2x(2x - 1) = 0$

Split the problem into two pieces and solve for x:

$2x = 0$ and $2x - 1 = 0$

$x = 0$ and $2x = 1 \rightarrow x = ½$

Method 2: $4x\left(x - \frac{1}{2}\right) = 0 \quad \rightarrow \quad x = 0 \text{ and } \frac{1}{2}$

Exercises in factoring and solving for x. Determine all the values of x which will satisfy these equations.

1	$x^2 + 3x = 0$	x(x+3)=0	x = 0 and -3	x =
2	$2x^2 + 6x = 0$	2x(x+3)=0	x = 0 and -3	x = -3 and...
3	$5x^2 + 5x = 0$	5x(x+5)=0	x = 0 and -5	x =
4	$x^2 - 3x = 0$	x(x-3)=0	x = 0 and 3	x =
5	$6x^2 - 3x = 0$	3x(2x–1)=0	x = 0 and 0.5	x = 0.5 and...
6	$3x^2 - 1x = 0$	3x(x – 1/3) = 0	x = 0 and 1/3	x =
7	$16x^2 - 4x = 0$	16x(x – ¼) = 0	x = 0 and ¼	x =
8	$25x^2 + 5x = 0$	25x(x + 1/5) = 0	x = 0 and -1/5	x =

Now things get tougher....

Example 4: $x^2 + 2x = 3$

Method 1: Factor out the x again.... $x(x + 2) = 3$. But now it's not so clear. That 3 is annoying. We could just guess... and probably realize that x can be 1. But could it also have another answer? Quadratics generally have 2 answers. Guessing this will be a bit more involved... but with some effort you can....the other answer is -3. x = 1 and -3.

Method 2: Bring everything to one side and factor it... like so.

$$x^2 + 2x = 3 \quad \rightarrow \quad x^2 + 2x - 3 = 0$$

Now factor it... $(x+\ ?\,)(x+\ ?\,) = x^2 + 2x - 3 = 0.$

...and just start guessing. Educated guessing.

The mystery terms on the right need to multiply to become –3. That means one will be negative and one positive. Start listing combinations that do this...

$$\mathbf{1 \cdot (-3) = -3} \qquad \textbf{and} \qquad \mathbf{-1 \cdot 3 = -3}$$

Let's test drive the first one....

$$(x + 1)(x - 3) \;=\; x^2 + 1x - 3x - 3 \;=\; x^2 - 2x - 3 \;\neq x^2 + 2x - 3$$

That didn't work.... but it was close. Let's try the other one....

$$(x - 1)(x + 3) \;=\; x^2 - 1x + 3x - 3 \;=\; x^2 + 2x - 3$$

That one worked.

$$(x - 1)(x + 3) = x^2 + 2x - 3 = 0$$

Now it is easy to see that if x = 1 or -3 the equation will be satisfied.

So the general method is that when you are factoring something into $(x+\ ?\,)(x+\ ?\,)$. The mystery terms must multiply together to produce the rightmost term of the quadratic, and they must also add together to produce the coefficion of the middle term. Here's what I mean.

Example 5: $x^2 + 2x - 8 = 0$. You want something that looks like this: $(x+\ ?\,)(x+\ ?\,) = 0$

Start by making a list of combinations that multiply to -8 and then also figure out how they add up.

$?_1$	$?_2$	multiply to	sum to
1	-8	-8	-7
-1	8	-8	7
2	-4	-8	-2
-2	**4**	**-8**	**2**

Then choose the combination that works. For our problem, $x^2 + 2x - 8$, the last combination works.

$$(x - 2)(x + 4) \;=\; x^2 - 2x + 4x - 8 \;=\; x^2 + 2x - 8 \;= 0$$

Ta da!

$$(x-2)(x+4)=0$$

Clearly if x = 2 or -4 the equation will be satisfied.

Now you try....

9	$x^2-x-2=0$ x = -1 and 2	x =
10	$x^2-x-12=0$ x = -3 and 4	x = -3 and
11	$x^2-5x-6=0$ x = -1 and 6	x =
12	$x^2+3x-10=0$ x = -5 and 2	x =
13	$x^2-5x+6=0$ x = 2 and 3	x = 2 and...
14	$x^2+5x+6=0$ x = -2 and -3	x =
15	$x^2+6x+5=0$ x = -1 and -5	x = -1 and...
16	$x^2-8x+15=0$ x = 3 and 5	x =

Essentially what you want to do is look at the quadratic equation in the form $x^2+bx+c=0$ and find values that multiply to become *c* and add to become *b*.

To this point we've been looking at equations are relatively easy to solve. But what if I make an equation that looks like this: $x^2-8x+3=0$. The table of possibilities is pretty limited...

$?_1$	$?_2$	multiply to	sum to
1	3	3	4
-1	-3	3	-4

unless we start looking at weirder ways to multiply to get 3...

$?_1$	$?_2$	multiply to	sum to
0.5	6	3	6.5
-1/3	-9	3	$-9.\overline{33}$

...the possibilities are endless.

So we bring out the big guns: The Quadratic Formula.

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Put your quadratic into this form: $ax^2 + bx + c = 0,$ like we've been doing, then solve for x using this famous formula.

Let's redo #14 using the quadratic formula: $x^2 + 5x + 6 = 0.$

It's already in the form $ax^2 + bx + c = 0.$

Just pick off the *a, b*, and *c* and throw them into the formula.

a = 1, b = 5, and c = 6.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2} = \frac{-5 \pm 1}{2}$$

$$x = \frac{-5 + 1}{2} \quad and \quad \frac{-5 - 1}{2}$$

$$x = \frac{-4}{2} \quad and \quad x = \frac{-6}{2}$$

$x = \boldsymbol{-2}$ ***and*** $\boldsymbol{-3}.$ Exactly what we got using the other method.

Now let's do the problem I suggested above which seemed too hard: $x^2 - 8x + 3 = 0.$

Pick off the *a, b*, and *c*. a = 1, b = -8, and c = 3. Plug these into the quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(3)}}{2(1)} = \frac{8 \pm \sqrt{64 - 12}}{2} = \frac{8 \pm \sqrt{52}}{2} \cong \frac{8 \pm 7.21 \ldots}{2}$$

$$x \cong \frac{8 \pm 7.21}{2} \cong \quad \boldsymbol{7.605 \ and \ 0.395.}$$

I never would have guessed this.

Reverse engineer it...

$$(x - 7.605\ )(x - 0.395) = 0$$

FOIL it out... to get the original equation.

$$x^2 - 7.605x - 0.395x + 3.004 = x^2 - 8x + 3.004 = 0.$$

And we're only off a few thousandths because we rounded $\sqrt{52}$ to 7.21.

Your turn....

Use the quadratic formula to find *x*.

9	$x^2 - x - 12 = 0$ x = -3 and 4	x =
10	$x^2 - 5x + 6 = 0$ x = 2 and 3	x =
11	$x^2 + 6x + 5 = 0$ x = -1 and -5	x =
12	$3x^2 + 8x + 5 = 0$ x = -1 and -5/3	x = -5/3 and...
13	$2x^2 - 7x + 3 = 0$ x = 3 and 1/2	x = 3 and...
14	$-2x^2 - 7x - 3 = 0$ x = -3 and -1/2	x =
15	$4x^2 - 10x + 5 = 0$ x ≅ 1.809 and 0.691	x ≅ 0.691 and...
16	$-8x^2 + 4x + 4 = 0$ x = -0.5 and 1	x =