BLC150 Name: ________________________________

HW-10- Linear Equations and Graphs, pt. II, plus!

The slope equation does a lot of work.
In fact, it does pretty much everything in the world of linear equations.
First of all it finds the slope of a line connecting 2 points.

$$\boldsymbol{m} = slope = \frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1} = \frac{y_1-y_2}{x_1-x_2}.$$

The input data is from two points, (x_1, y_1) and (x_2, y_2).

But it can also be rearranged to become the "Point-Slope" formula,

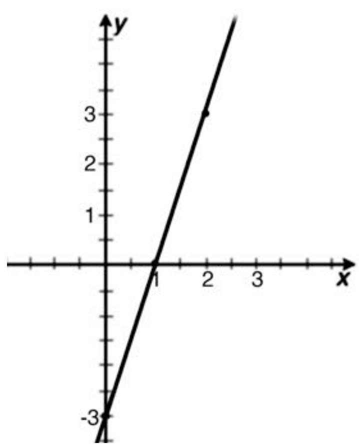
which is a machine that cranks out $y = mx + b$ formulae.

$$\frac{(rise)}{(run)} = m \quad \rightarrow \quad (rise) = m(run) \quad = \quad (y_2 - y_1) = m(x_2 - x_1)$$

Typically it is written like this: $(y - y_1) = m(x - x_1)$

Here's an example of how to use the Point-Slope formula.

Let's say you know a point, (x, y), and a slope, m. Find the $y = mx + b$ form for the equation of this line.	$(2, 3)$ is the point and $m = 3$ is the slope.
Simply plug what you know into the point-slope formula.	$(y - y_1) = m(x - x_1)$ $(y - 3) = 3(x - 2)$
Solve this equation for y and you're done. Ta da!	$y - 3 = 3x - 6 \quad \rightarrow \quad y = 3x - 3$



Here's the graph of this equation.

There is another way to do this in which you simply plug the point and the slope into the $y = mx + b$ formula and solve for b. Like this... $y = mx + b \quad \rightarrow \quad 3 = 3(2) + b \quad \rightarrow \quad b = -3$.
Once you establish what b is, then put it all back together: $y = 3x - 3$

Exercise 1: Figure out the equation of the line in $y = mx + b$ form using the information provided and then graph it. [Label axes and add some numbers to show scale.]

E.g. Given two points: (2, 3) and (0, –3)

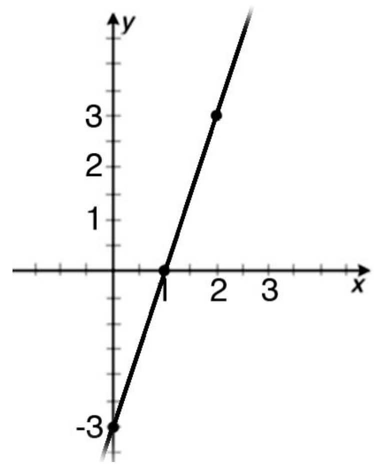
First find the slope.

$$\boldsymbol{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-3)}{2-0} = \frac{6}{2} = 3$$

Then plug the slope and one of your known points into the point-slope equation

$$(y - y_1) = m(x - x_1) \rightarrow (y - 3) = 3(x - 2) \rightarrow \boldsymbol{y = 3x - 3}$$

Then graph it.



1. (3, 3) and (1, 2) Do this just like the example above.

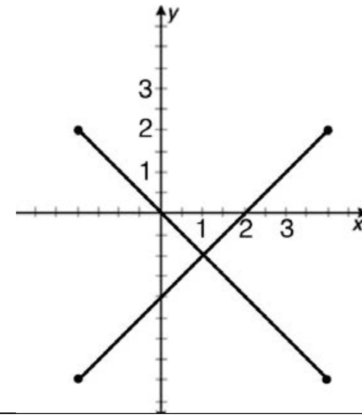
2. m = 3/2 and (1, 2) Go straight to the point-slope formula and plug in the given information... solve for y.

3. m = -4 and (1, 4) Like the previous problem, go straight to the point-slope formula.

4. (-2, 2) and (4, -4)

Also figure out the line between these two points: (4, 2) and (-2, -4)

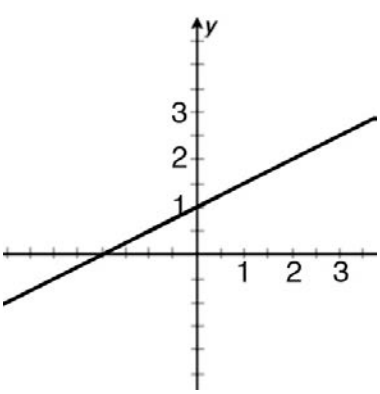
Graph both on the same graph. [I did it for you.]



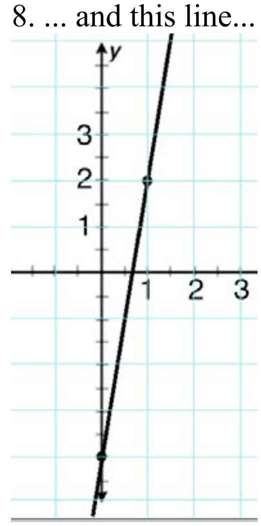
5. (-4, 2) and (3, 2) Tricky. I suggest graphing this first so that you see what's going on, then do the math.

6. (2, -4) and (2, 3) More Tricky. Again... graph it first so that you see what's happing, then do the math.

7. Figure out the equation of this line by eye. Find a y-intercept and a slope and put it into *y* = *mx* + *b*.



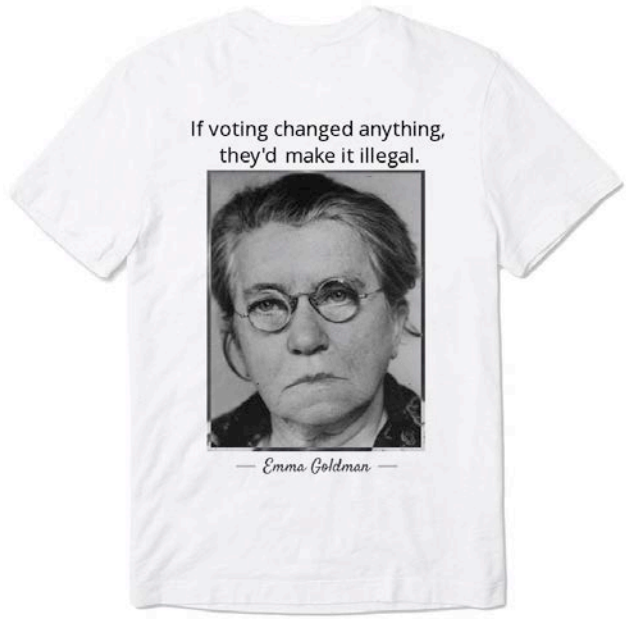
8. ... and this line...



Exercise 2: A few word problems.

2.1 You are the leader of an anarchist group on campus. (Ironic, no?) You want to have some tee-shirts printed up for a march against the World Bank and the IMF in DC. The tee-shirt store says that there is a $40 initial fee for making the silk screen and then it will charge $12 for every tee-shirt. So, for example, if you just get 1 tee-shirt, you will have to pay $52. Two tee-shirts will cost $64... etc. Write a linear equation in the form y = mx + b to describe this situation. The y-term will be the amount of money you will have to pay the printer and the x-term will be the number of tee-shirts you buy. The b will be the initial up-front money. Then draw a thumbnail graph of this linear equation to the right. If you have trouble with this try plotting a bunch of points in this form: (x, y) → (number of shirts, cost)

(1, $52), (2, $64), etc.



2.2 You are in a van headed for a Grateful Dead concert in Cleveland. Your driver is going 60 miles per hour. Assuming you are able to continue going 60mph, write an equation that will output how far you have travelled as a function of time? The y-term will be the total distance travelled. The x-term will be hours traveled. The slope will be 60mph. Sketch a quick graph.

2.3 You found a new passion: photography. Not digital photography.... real darkroom-and-chemicals photography. The problem? The set-up cost is expensive. You need a camera (with various accessories) and darkroom equipment. At Hoffman's Bard Store in Red Hook you found an old steamer trunk with everything you would ever need. A Nikon Nikkormat with a 22mm and 50mm lens, tripod, plunger, enlarger, chemical trays, a timer, negative processing contraption, and even red light bulbs with fixtures. Everything you might want.   All for $250. You also look into the cost of film and chemicals for the darkroom and figure that all tolled, a roll of film and the chemicals and papers necessary to process and print it will cost $18 per roll. Because estimates such as these are almost always underestimates, you wisely round your estimate up to $20 per roll. That's essentially the operating cost of pursuing photography. An initial, up-front investiment of $250, and an operating cost of $20 per roll.
Write an equation which outputs y, the total investiment in this endeavour, and inputs x, the number of rolls of film you will buy and process.

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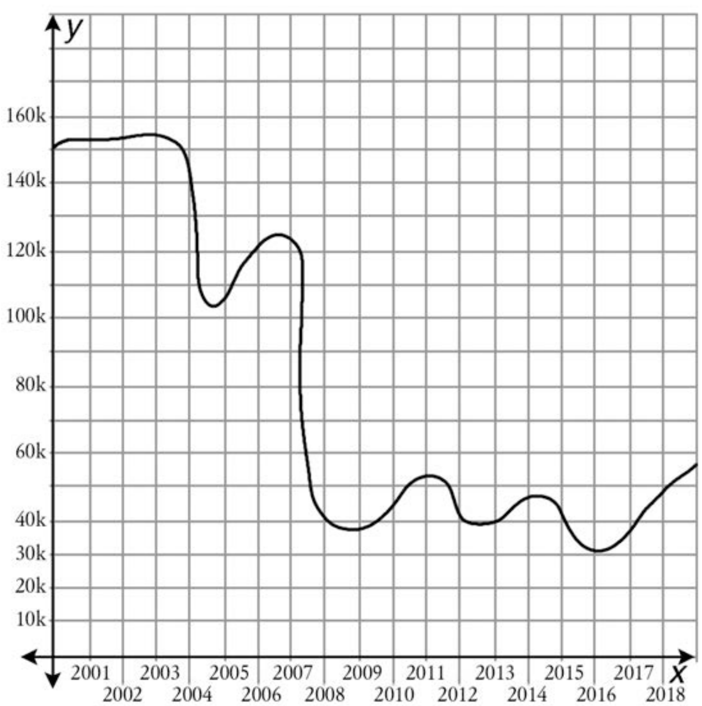
3. This graph is the population of a town over time. Write up a quick story to go along with this graph.



Area density:

Let's say you are part of an archeological dig and you have excavated a 1x1 meter region. At depth level 3 you found 7 coins. At level 2 you found 5 coins and at level 1 you found 2 coins. This data and their densities are as shown in table 1.

TABLE 1	Artifacts found in excavation area of 1 square meter.	Artifact Denisty
Level 1	2 coins	2/sq-meter (2 per square meter)
Level 2	5 coins	5/sq-meter
Level 2	7 coins	7/sq-meter

Now let's say you received new data from another archeological team working on a different region. They report finding 5 coins at level 3, 4 coins at level 2, and 1 coin at level 1, but their area of excavation was just 0.5 x 0.4 meters. The area of this region is rectangular, but its area in square meters is easy to figure out. Just length times width.

$$length \times width = 0.5 \cdot 0.4 = 0.2\ sq.meters$$

You want this new data to be comparable to the data from the previous report (Table 1). You need to compare the densities... artifacts per square meter. 1 coin per 0.2 square meters needs to be converted to coins per square meter like so: $\frac{1\ coin}{0.2\ sq.m} = \frac{5\ coins}{sq.m}$. [Literally $\frac{1}{0.2} = 5$] Makes sense. Think about it. 0.2 square meters is one fifth of a square meter. So multiply top and bottom by 5 to figure out the artifacts per square meter.

TABLE 2	Artifacts found in excavation area of 0.2 square meters.	Artifact Denisty
Level 1	1 coins	5/sq-meter
Level 2	4 coins	20/sq-meter
Level 2	5 coins	25/sq-meter

Now you can compare artifact densities since they are calibrated to be the same area.

Exercise:

You have three reports of coins being found from three different locations excavated by a different team, but each location was digging out a different sized region. Here is the raw data. Figure out the densities of coins per square meter for the three levels of each site.

Team 1: 3 x 5 meter region excavated		
	Artifacts found	Density (coins per sq. meter)
Level 1	28	ca. 1.87 per square meter
Level 2	15	
Level 3	2	

Team 2: 1 x 3 meter region excavated		
	Artifacts found	Density (coins per sq. meter)
Level 1	6	
Level 2	4	ca. 1.3 per square meter
Level 3	0	

Team 3: 4 x 4 meter region excavated		
	Artifacts found	Density (coins per sq. meter)
Level 1	32	
Level 2	16	
Level 3	3	ca. 0.2 per square meter